THE DYNAMIC LUENBERGER-HICKS-MOORSTEEN PRODUCTIVITY INDICATOR WITH AN APPLICATION TO DAIRY FARMS IN SOUTH WEST ENGLAND

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Abstract. This paper introduces a dynamic Luenberger-Hicks-Moorsteen (LHM) productivity indicator that takes into account the adjustment costs of changing the level of quasi-fixed capital inputs. Being additively complete in the dynamic sense, the LHM indicator is decomposed into contributions of outputs, variable inputs and investments in dynamic factors. Moreover, we decompose the LHM indicator into technical change, technical inefficiency change and scale inefficiency change using an investment-, output- and input-direction. Employing a nonparametric framework, the empirical application focuses on the dairy sector in South West England over the period 2001 – 2014.
Keywords Luenberger-Hicks-Moorsteen indicator, productivity growth, adjustment cost, additive completeness
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1. Introduction

Productivity analysis is an important monitoring tool of economic performance. An area having only received scant attention is the appropriate modeling of intertemporal linkages of production decisions. The vast majority of studies use a *static* approach, in which firms are assumed to instantaneously change the levels of all types of inputs to their long-run equilibrium. Yet, this misrepresents the sluggish adjustment of quasi-fixed inputs. Investment in quasi-fixed inputs is needed for improving productivity in the long run, but its associated adjustment costs may lead to an misestimation of productivity.

Only few studies develop *dynamic* productivity measures that appropriately represent the adjustment costs associated with investment. Starting from a dynamic profit-maximization problem, Luh and Stefanou (1991) assess dynamic productivity growth as the Solow residual growth in outputs not explained by growth in variable and quasi-fixed inputs and changing shadow values of the input stock. The framework also allows assessment of the contribution of technical change and scale change. Rungsuriyawiboon and Stefanou (2008) extend this framework by accounting for technical efficiency change starting from a dynamic cost-minimization problem. Oude Lansink, Stefanou, and Serra (2015) develop a dynamic Luenberger productivity indicator that is based on dynamic directional distance functions. This measure adapts the static Luenberger productivity indicator of Chambers (2002) to a dynamic context.

The dynamic Solow-residual-based productivity measures of Luh and Stefanou (1991) and Rungsuriyawiboon and Stefanou (2008) can only be estimated parametrically. They also depend on some behavioral economic assumption. However, the parametric approach inherently relies on the functional specification underlying the production technology. In addition, the behavioral economic assumption can be unclear in various instances. Oude Lansink, Stefanou, and Serra (2015)'s dynamic Luenberger productivity indicators can be estimated both parametrically (as in their study) and nonparametrically (as in Kapelko, Oude Lansink, and Stefanou (2015)). Moreover, since it is possible to compute primal as well as dual dynamic productivity measures, one could solely focus on the primal representation, withholding the dual behavioral assumption. However, the dynamic Luenberger indicator is not "additively complete" (O'Donnell (2012)) in the dynamic sense in that it cannot be decomposed into components of output growth, input decline and investment expansion. This prevents us to give concrete recommendations about *how* to improve dynamic productivity.

The current paper develops a dynamic Luenberger-Hicks-Moorsteen (LHM) productivity indicator. The static LHM indicator developed by Briec and Kerstens (2004) is additively complete in the static sense and can thus be expressed as the difference between an output aggregator and an input aggregator. Ang and Kerstens (2017a) show that the static LHM indicator can be decomposed into contributions of technical change, technical inefficiency change and scale inefficiency change.

Our newly developed dynamic LHM indicator is additively complete in the dynamic sense and can thus be decomposed into contributions of outputs, variable inputs and investments. Furthermore, in line with Ang and Kerstens (2017a), it can also be decomposed into technical change, technical inefficiency change and scale inefficiency change.

Using a nonparametric framework, the empirical application focuses on the dairy sector in South West England covering the years 2001 − 2014. South West England is the most dairy-intensive region in England. The considered period represents the late phase of the recently expired milk quota system, which held for all members of the European Union from 1984 to 2015. In 2003, it was announced that the milk quota system would be abolished, leading to investment expansion of proactive farmers. In the subsequent period, the milk quotum was progressively increased at several instances, which made it effectively non-binding for English dairy farms. We expect that the changing policy circumstances have an impact on the contributions of outputs, inputs and dynamic factors, on the one hand, and technical change, technical inefficiency change and scale inefficiency change, on the other hand. This makes our newly developed dynamic LHM indicator and its various decompositions useful for this application.

This paper is structured as follows. The next section develops the dynamic LHM indicator and describes the empirical application, respectively. This is followed by a presentation of the results. The final section concludes.

2. The dynamic Luenberger-Hicks-Moorsteen indicator

Let $\mathbf{X}_t \in \mathbb{R}^n_+$ be the inputs, $\mathbf{Y}_t \in \mathbb{R}^m_+$ outputs, $\mathbf{I}_t \in \mathbb{R}^o_+$ investment and $\mathbf{K}_t \in \mathbb{R}^o_+$ the associated capital stock. The dynamic technology set is defined as follows (Ang and Oude Lansink, 2016; Silva, Oude Lansink, and Stefanou, 2015):[1]

$$\boldsymbol{\mathcal{T}}_t(\mathbf{K}_t) = \left\{(\mathbf{X}_t, \mathbf{I}_t, \mathbf{Y}_t) \in \mathbb{R}^{n+m+o}_+ | (\mathbf{X}_t, \mathbf{I}_t) \text{ produces } \mathbf{Y}_t \text{ given } \mathbf{K}_t\right\}$$

In line with Silva, Oude Lansink, and Stefanou (2015) and Ang and Oude Lansink (2016), we make the following assumptions regarding the dynamic technology:

Axiom 1 (Closedness). $\boldsymbol{\mathcal{T}}_t(\mathbf{K}_t)$ *is closed.*

Axiom 2 (Free disposability of inputs and outputs). *if* $(\mathbf{X}'_t, -\mathbf{Y}'_t) \geq (\mathbf{X}_t, -\mathbf{Y}_t)$ *then* $(\mathbf{X}_t, \mathbf{I}_t, \mathbf{Y}_t) \in \boldsymbol{\mathcal{T}}_t(\mathbf{K}_t) \Rightarrow (\mathbf{X}'_t, \mathbf{I}_t, \mathbf{Y}'_t) \in \boldsymbol{\mathcal{T}}_t(\mathbf{K}_t)$.

Axiom 3 (Investment inaction is possible). $(\mathbf{X}_t, \mathbf{0}_t, \mathbf{Y}_t) \in \boldsymbol{\mathcal{T}}_t(\mathbf{K}_t)$.

Axiom 4 (Negative monotonicity in investments). *if* $\mathbf{I}_t \geq \mathbf{I}'_t$ *then* $(\mathbf{X}_t, \mathbf{I}_t, \mathbf{Y}_t) \in \boldsymbol{\mathcal{T}}_t(\mathbf{K}_t) \Rightarrow (\mathbf{X}_t, \mathbf{I}'_t, \mathbf{Y}_t) \in \boldsymbol{\mathcal{T}}_t(\mathbf{K}_t)$.

[1] Silva, Oude Lansink, and Stefanou (2015) build on the dynamic input directional distance function framework. Ang and Oude Lansink (2016) generalize this to a dynamic directional distance function framework, which also considers inefficiencies in the output direction. This paper follows the latter approach.

Axiom 5 (Reverse nestedness in capital stock)**.** *if* $\mathbf{K}_t' \geq \mathbf{K}_t$ *then* $\boldsymbol{\mathcal{T}}_t(\mathbf{K}_t) \subseteq \boldsymbol{\mathcal{T}}_t(\mathbf{K}_t')$.

Axiom 6 (Convexity)**.** *Technology set* $\boldsymbol{\mathcal{T}}_t(\mathbf{K}_t)$ *is convex.*

Axiom 3 states that production is possible without investment and is consistent with periodic observed investment spikes found in the empirical literature. Adjustment costs are modeled by Axiom 4 and Axiom 6 as a (temporary) reduction of output as a consequence of investment. Finally, Axiom 5 models that an addition in the capital stock widens the production possibilities.

The dynamic technology can equivalently be represented by the dynamic directional distance function (Ang and Oude Lansink, 2016; Silva, Oude Lansink, and Stefanou, 2015). The time-related dynamic directional distance function for $(a,b) \in \{t, t+1\} \times \{t, t+1\}$ is:

(1)

$$D_b(\mathbf{X}_a, \mathbf{I}_a, \mathbf{Y}_a; \mathbf{g}_a | \mathbf{K}_b) = \sup \left\{ \beta \in \mathbb{R} : (\mathbf{X}_a - \beta \mathbf{g}_a^x, \mathbf{I}_a + \beta \mathbf{g}_a^i, \mathbf{Y}_a + \beta \mathbf{g}_a^y) \in \boldsymbol{\mathcal{T}}_b(\mathbf{K}_b) \right\},$$

if $(\mathbf{X}_a - \beta \mathbf{g}_a^x, \mathbf{I}_a + \beta \mathbf{g}_a^i, \mathbf{Y}_a + \beta \mathbf{g}_a^y) \in \boldsymbol{\mathcal{T}}_b(\mathbf{K}_b)$ for some β and $D_b(\mathbf{X}_a, \mathbf{I}_a, \mathbf{Y}_a; \mathbf{g}_a | \mathbf{K}_b) = -\infty$ otherwise. Here, $\mathbf{g}_a = (\mathbf{g}_a^x, \mathbf{g}_a^i, \mathbf{g}_a^y) \in \mathbb{R}_{++}^{n+m+o}$ represents the directional vector.

O'Donnell (2012) defines additive completeness of Total Factor Productivity (TFP) in the static sense. We extend this to the dynamic context:

Definition 1 (additive completeness in the dynamic sense)**.** *Let* $TFPI_s(\mathbf{X}_t, \mathbf{I}_t, \mathbf{Y}_t, \mathbf{X}_s, \mathbf{I}_s, \mathbf{Y}_s)$ *denote an index number that compares TFP in period s with TFP in period t using s as a base.* $TFPI_s(\mathbf{X}_t, \mathbf{I}_t, \mathbf{Y}_t, \mathbf{X}_s, \mathbf{I}_s, \mathbf{Y}_s)$ *is additively complete in the dynamic sense if and only if it can be expressed in the form*

$$TFPI_s(\mathbf{X}_t, \mathbf{I}_t, \mathbf{Y}_t, \mathbf{X}_s, \mathbf{I}_s, \mathbf{Y}_s) = \mathcal{Y}(\mathbf{Y}_t) - \mathcal{Y}(\mathbf{Y}_s) + \mathcal{I}(\mathbf{I}_t) - \mathcal{I}(\mathbf{I}_s) - \mathcal{X}(\mathbf{X}_t) + \mathcal{X}(\mathbf{X}_s)$$

$$\equiv LY_{t,t+1} + LI_{t,t+1} - LX_{t,t+1}$$

where $\mathcal{Y}(\cdot), \mathcal{I}(\cdot)$ *and* $\mathcal{X}(\cdot)$ *are non-negative non-decreasing functions satisfying the translation property* $\Upsilon(\boldsymbol{\Upsilon} + \lambda \boldsymbol{\Upsilon}) = \Upsilon(\boldsymbol{\Upsilon}) + \lambda$.

Intuitively, additive completeness in the dynamic sense means that productivity can be decomposed in three components: output change $LY_{t,t+1}$, investment change $LI_{t,t+1}$ and input change $LX_{t,t+1}$. Recently, Oude Lansink, Stefanou, and Serra (2015) proposed a dynamic Luenberger indicator. The Luenberger indicator is not additively complete and cannot be decomposed into these three components. Ang and Kerstens (2017a) show that the static Luenberger-Hicks-Moorsteen (LHM) indicator of Briec and Kerstens (2004) is additively complete in the static sense. This paper develops a dynamic LHM indicator being additively complete in the dynamic sense. We define the dynamic LHM indicator with base period t as:

$$
\begin{aligned}
(2)\qquad & LHM_t(\mathbf{X}_{t+1}, \mathbf{I}_{t+1}, \mathbf{Y}_{t+1}, \mathbf{X}_t, \mathbf{I}_t, \mathbf{Y}_t; \mathbf{g}_t, \mathbf{g}_{t+1}) \\
& = (D_t(\mathbf{X}_t, \mathbf{I}_t, \mathbf{Y}_t; (0, 0, \mathbf{g}_t^y)|\mathbf{K}_t) - D_t(\mathbf{X}_t, \mathbf{I}_t, \mathbf{Y}_{t+1}; (0, 0, \mathbf{g}_{t+1}^y)|\mathbf{K}_t)) \\
& - (D_t(\mathbf{X}_{t+1}, \mathbf{I}_t, \mathbf{Y}_t; (\mathbf{g}_{t+1}^x, 0, 0)|\mathbf{K}_t) - D_t(\mathbf{X}_t, \mathbf{I}_t, \mathbf{Y}_t; (\mathbf{g}_t^x, 0, 0)|\mathbf{K}_t)) \\
& + (D_t(\mathbf{X}_t, \mathbf{I}_t, \mathbf{Y}_t; (0, \mathbf{g}_t^i, 0)|\mathbf{K}_t) - D_t(\mathbf{X}_t, \mathbf{I}_{t+1}, \mathbf{Y}_t; (0, \mathbf{g}_{t+1}^i, 0)|\mathbf{K}_t)) \\
& \equiv LY_t(\mathbf{X}_t, \mathbf{I}_t, \mathbf{Y}_t, \mathbf{Y}_{t+1}; \mathbf{g}_t^y, \mathbf{g}_{t+1}^y) - LX_t(\mathbf{X}_t, \mathbf{X}_{t+1}, \mathbf{I}_t, \mathbf{Y}_t; \mathbf{g}_t^x, \mathbf{g}_{t+1}^x) \\
& + LI_t(\mathbf{X}_t, \mathbf{I}_t, \mathbf{I}_{t+1}, \mathbf{Y}_t; \mathbf{g}_t^i, \mathbf{g}_{t+1}^i),
\end{aligned}
$$

Analogously, a base period $t + 1$ dynamic LHM indicator is defined as:

(3)

$$
\begin{aligned}
& LHM_{t+1}(\mathbf{X}_{t+1}, \mathbf{I}_{t+1}, \mathbf{Y}_{t+1}, \mathbf{X}_t, \mathbf{I}_t, \mathbf{Y}_t; \mathbf{g}_t, \mathbf{g}_{t+1}) \\
& = (D_{t+1}(\mathbf{X}_{t+1}, \mathbf{I}_{t+1}, \mathbf{Y}_t; (0, 0, \mathbf{g}_t^y)|\mathbf{K}_{t+1}) - D_{t+1}(\mathbf{X}_{t+1}, \mathbf{I}_{t+1}, \mathbf{Y}_{t+1}; (0, 0, \mathbf{g}_{t+1}^y)|\mathbf{K}_{t+1})) \\
& - (D_{t+1}(\mathbf{X}_{t+1}, \mathbf{I}_{t+1}, \mathbf{Y}_{t+1}; (\mathbf{g}_{t+1}^x, 0, 0)|\mathbf{K}_{t+1}) - D_{t+1}(\mathbf{X}_t, \mathbf{I}_{t+1}, \mathbf{Y}_{t+1}; (\mathbf{g}_t^x, 0, 0)|\mathbf{K}_{t+1})) \\
& + (D_{t+1}(\mathbf{X}_{t+1}, \mathbf{I}_t, \mathbf{Y}_{t+1}; (0, \mathbf{g}_t^i, 0)|\mathbf{K}_{t+1}) - D_{t+1}(\mathbf{X}_{t+1}, \mathbf{I}_{t+1}, \mathbf{Y}_{t+1}; (0, \mathbf{g}_{t+1}^i, 0)|\mathbf{K}_{t+1})) \\
& \equiv LY_{t+1}(\mathbf{X}_{t+1}, \mathbf{I}_{t+1}, \mathbf{Y}_{t+1}, \mathbf{Y}_t; \mathbf{g}_t^y, \mathbf{g}_{t+1}^y) - LX_{t+1}(\mathbf{X}_t, \mathbf{X}_{t+1}, \mathbf{I}_{t+1}, \mathbf{Y}_{t+1}; \mathbf{g}_t^x, \mathbf{g}_{t+1}^x) \\
& + LI_{t+1}(\mathbf{X}_{t+1}, \mathbf{I}_t, \mathbf{I}_{t+1}, \mathbf{Y}_{t+1}; \mathbf{g}_t^i, \mathbf{g}_{t+1}^i),
\end{aligned}
$$

Finally, one takes an arithmetic mean of LHM_t and LHM_{t+1} to avoid an arbitrary choice of base periods:

$$
\begin{aligned}
(4)\qquad & LHM_{t,t+1}(\mathbf{X}_t, \mathbf{I}_t, \mathbf{Y}_t, \mathbf{X}_{t+1}, \mathbf{I}_{t+1}, \mathbf{Y}_{t+1}; \mathbf{g}_t, \mathbf{g}_{t+1}) \\
& = \frac{1}{2}\big[LHM_t(\mathbf{X}_{t+1}, \mathbf{I}_{t+1}, \mathbf{Y}_{t+1}, \mathbf{X}_t, \mathbf{I}_t, \mathbf{Y}_t; \mathbf{g}_t, \mathbf{g}_{t+1}) \\
& + LHM_{t+1}(\mathbf{X}_{t+1}, \mathbf{I}_{t+1}, \mathbf{Y}_{t+1}, \mathbf{X}_t, \mathbf{I}_t, \mathbf{Y}_t; \mathbf{g}_t, \mathbf{g}_{t+1})\big]
\end{aligned}
$$

The dynamic LHM indicator takes into account the adjustment costs associated with investments. It is straightforward to verify that the dynamic LHM is additively complete in the dynamic sense. This allows us to analyze the extent to which input growth, investment growth and output growth contribute to dynamic productivity growth.

3. Decomposition of the dynamic Luenberger-Hicks-Moorsteen indicator

This paper decomposes the dynamic LHM indicator into dynamic technical change, dynamic technical inefficiency change and dynamic scale inefficiency change using the investment direction. In the remainder of the paper, we omit "dynamic" in each component for brevity. In line with the decomposition of the static LHM indicator in Ang and Kerstens (2017a), the dynamic LHM indicator can be decomposed using investment directions, output directions or variable input directions. This section only focuses on the decomposition using the investment direction.

Nonetheless, we also decompose the dynamic LHM indicator using the output direction and input direction as explained in Ang and Kerstens (2017a). Our decomposition consists of three components:[2]

$$LHM_{t,t+1} = \Delta T^i_{t,t+1} + \Delta TEI^i_{t,t+1} + \Delta SEC^i_{t,t+1}, \tag{5}$$

representing technical change, technical inefficiency change and scale inefficiency change, respectively. The definitions of the components are similar to the components of the static LHM indicator in Ang and Kerstens (2017a), except for a subtle difference in the scale inefficiency change component. In what follows, we therefore solely focus on $\Delta SEC^i_{t,t+1}$ and refer to the Appendix for the other components' definitions.

The scale inefficiency change component is obtained as the residual:

(6)

$$\begin{aligned}
LHM_{t,t+1} - \Delta T^i_{t,t+1} - \Delta TEI^i_{t,t+1} \\
= \frac{1}{2} \Big\{ & \big[D_t(\mathbf{X}_t, \mathbf{I}_t, \mathbf{Y}_t; (0,0,\mathbf{g}^y_t)|\mathbf{K}_t) - D_t(\mathbf{X}_t, \mathbf{I}_t, \mathbf{Y}_{t+1}; (0,0,\mathbf{g}^y_{t+1})|\mathbf{K}_t) \big] \\
& + \big[D_{t+1}(\mathbf{X}_{t+1}, \mathbf{I}_{t+1}, \mathbf{Y}_t; (0,0,\mathbf{g}^y_t)|\mathbf{K}_{t+1}) - D_{t+1}(\mathbf{X}_{t+1}, \mathbf{I}_{t+1}, \mathbf{Y}_{t+1}; (0,0,\mathbf{g}^y_{t+1})|\mathbf{K}_{t+1}) \big] \Big\} \\
- \frac{1}{2} \Big\{ & \big[D_t(\mathbf{X}_{t+1}, \mathbf{I}_t, \mathbf{Y}_t; (\mathbf{g}^x_{t+1},0,0)|\mathbf{K}_t) - D_t(\mathbf{X}_t, \mathbf{I}_t, \mathbf{Y}_t; (\mathbf{g}^x_t,0,0)|\mathbf{K}_t) \big] \\
& + \big[D_{t+1}(\mathbf{X}_{t+1}, \mathbf{I}_{t+1}, \mathbf{Y}_{t+1}; (\mathbf{g}^x_{t+1},0,0)|\mathbf{K}_{t+1}) - D_{t+1}(\mathbf{X}_t, \mathbf{I}_{t+1}, \mathbf{Y}_{t+1}; (\mathbf{g}^x_t,0,0)|\mathbf{K}_{t+1}) \big] \Big\} \\
+ \frac{1}{2} \Big\{ & \big[D_t(\mathbf{X}_{t+1}, \mathbf{I}_{t+1}, \mathbf{Y}_{t+1}; (0,\mathbf{g}^i_{t+1},0)|\mathbf{K}_t) - D_t(\mathbf{X}_t, \mathbf{I}_{t+1}, \mathbf{Y}_t; (0,\mathbf{g}^i_{t+1},0)|\mathbf{K}_t) \big] \\
& + \big[D_{t+1}(\mathbf{X}_{t+1}, \mathbf{I}_t, \mathbf{Y}_{t+1}; (0,\mathbf{g}^i_t,0)|\mathbf{K}_{t+1}) - D_{t+1}(\mathbf{X}_t, \mathbf{I}_t, \mathbf{Y}_t; (0,\mathbf{g}^i_t,0)|\mathbf{K}_{t+1}) \big] \Big\} .
\end{aligned}$$

In order to better understand the intuition behind what this exactly measures, we rewrite it in a more comprehensible form in analogy to Ang and Kerstens (2017a). First, define the projections of $\mathbf{I}_t$ and $\mathbf{I}_{t+1}$ on the production frontier at time t:

$$\mathbf{I}^*_t = \mathbf{I}_t + D_t(\mathbf{X}_t, \mathbf{I}_t, \mathbf{Y}_t; (0,\mathbf{g}^i_t,0)|\mathbf{K}_t)\mathbf{g}^i_t \tag{7a}$$

$$\mathbf{I}^{**}_{t+1} = \mathbf{I}_{t+1} + D_t(\mathbf{X}_{t+1}, \mathbf{I}_{t+1}, \mathbf{Y}_{t+1}; (0,\mathbf{g}^i_{t+1},0)|\mathbf{K}_t)\mathbf{g}^i_{t+1} \tag{7b}$$

Similarly, define the projections of $\mathbf{I}_t$ and $\mathbf{I}_{t+1}$ on the production frontier at time $t+1$:

$$\mathbf{I}^{**}_t = \mathbf{I}_t + D_{t+1}(\mathbf{X}_t, \mathbf{I}_t, \mathbf{Y}_t; (0,\mathbf{g}^i_t,0)|\mathbf{K}_{t+1})\mathbf{g}^i_t \tag{8a}$$

$$\mathbf{I}^*_{t+1} = \mathbf{I}_{t+1} + D_{t+1}(\mathbf{X}_{t+1}, \mathbf{I}_{t+1}, \mathbf{Y}_{t+1}; (0,\mathbf{g}^i_{t+1},0)|\mathbf{K}_{t+1})\mathbf{g}^i_{t+1} \tag{8b}$$

Adding and subtracting $D_t(\mathbf{X}_t, \mathbf{I}_t, \mathbf{Y}_t; (0,\mathbf{g}^i_t,0)|\mathbf{K}_t)$ and $D_{t+1}(\mathbf{X}_{t+1}, \mathbf{I}_{t+1}, \mathbf{Y}_{t+1}; (0,\mathbf{g}^i_{t+1},0)|\mathbf{K}_{t+1})$ to and from (6), respectively, and using the translation property of the directional

[2]Since we use directional distance functions as aggregator functions for outputs, investments and inputs, there is no mix inefficiency change (O'Donnell, 2012)

distance function and the definitions of the projections above, we find the scale inefficiency change component:

(9)
$$\begin{aligned}\Delta SEC^i_{t,t+1} = \frac{1}{2}\big\{ & \big[D_t(\mathbf{X}_t,\mathbf{I}_t,\mathbf{Y}_t;(0,0,\mathbf{g}^y_t)|\mathbf{K}_t) - D_t(\mathbf{X}_t,\mathbf{I}_t,\mathbf{Y}_{t+1};(0,0,\mathbf{g}^y_{t+1})|\mathbf{K}_t)\big] \\ & + \big[D_{t+1}(\mathbf{X}_{t+1},\mathbf{I}_{t+1},\mathbf{Y}_t;(0,0,\mathbf{g}^y_t)|\mathbf{K}_{t+1}) - D_{t+1}(\mathbf{X}_{t+1},\mathbf{I}_{t+1},\mathbf{Y}_{t+1};(0,0,\mathbf{g}^y_{t+1})|\mathbf{K}_{t+1})\big]\big\} \\ -\frac{1}{2}\big\{ & \big[D_t(\mathbf{X}_{t+1},\mathbf{I}_t,\mathbf{Y}_t;(\mathbf{g}^x_{t+1},0,0)|\mathbf{K}_t) - D_t(\mathbf{X}_t,\mathbf{I}_t,\mathbf{Y}_t;(\mathbf{g}^x_t,0,0)|\mathbf{K}_t)\big] \\ & + \big[D_{t+1}(\mathbf{X}_{t+1},\mathbf{I}_{t+1},\mathbf{Y}_{t+1};(\mathbf{g}^x_{t+1},0,0)|\mathbf{K}_{t+1}) - D_{t+1}(\mathbf{X}_t,\mathbf{I}_{t+1},\mathbf{Y}_{t+1};(\mathbf{g}^x_t,0,0)|\mathbf{K}_{t+1})\big]\big\} \\ +\frac{1}{2}\big\{ & \big[D_t(\mathbf{X}_t,\mathbf{I}^*_t,\mathbf{Y}_t;(0,\mathbf{g}^i_t,0)|\mathbf{K}_t) - D_t(\mathbf{X}_t,\mathbf{I}^{**}_{t+1},\mathbf{Y}_t;(0,\mathbf{g}^i_{t+1},0)|\mathbf{K}_t)\big] \\ & + \big[D_{t+1}(\mathbf{X}_{t+1},\mathbf{I}^{**}_t,\mathbf{Y}_{t+1};(0,\mathbf{g}^i_t,0)|\mathbf{K}_{t+1}) - D_{t+1}(\mathbf{X}_{t+1},\mathbf{I}^*_{t+1},\mathbf{Y}_{t+1};(0,\mathbf{g}^i_{t+1},0)|\mathbf{K}_{t+1})\big]\big\} \\ \equiv \frac{1}{2} & \left\{\Delta SEC^i_t + \Delta SEC^i_{t+1}\right\},\end{aligned}$$

which has the interpretation of measuring changes in "global" returns to scale in line with Diewert and Fox (2014, 2017) and Ang and Kerstens (2017a). The scale inefficiency change component is the arithmetic average of a Laspeyres type indicator

(10)
$$\begin{aligned}\Delta SEC^i_t = & \frac{1}{4}\big\{\big[D_t(\mathbf{X}_t,\mathbf{I}_t,\mathbf{Y}_t;(0,0,\mathbf{g}^y_t)|\mathbf{K}_t) - D_t(\mathbf{X}_t,\mathbf{I}_t,\mathbf{Y}_{t+1};(0,0,\mathbf{g}^y_{t+1})|\mathbf{K}_t)\big] \\ & - \big[D_t(\mathbf{X}_{t+1},\mathbf{I}_t,\mathbf{Y}_t;(\mathbf{g}^x_{t+1},0,0)|\mathbf{K}_t) - D_t(\mathbf{X}_t,\mathbf{I}_t,\mathbf{Y}_t;(\mathbf{g}^x_t,0,0)|\mathbf{K}_t)\big]\big\} \\ & + \frac{1}{4}\big\{\big[D_t(\mathbf{X}_t,\mathbf{I}_t,\mathbf{Y}_t;(0,0,\mathbf{g}^y_t)|\mathbf{K}_t) - D_t(\mathbf{X}_t,\mathbf{I}_t,\mathbf{Y}_{t+1};(0,0,\mathbf{g}^y_{t+1})|\mathbf{K}_t)\big] \\ & + \big[D_t(\mathbf{X}_t,\mathbf{I}^*_t,\mathbf{Y}_t;(0,\mathbf{g}^i_t,0)|\mathbf{K}_t) - D_t(\mathbf{X}_t,\mathbf{I}^{**}_{t+1},\mathbf{Y}_t;(0,\mathbf{g}^i_{t+1},0)|\mathbf{K}_t)\big]\big\} \\ & + \frac{1}{4}\big\{\big[D_t(\mathbf{X}_t,\mathbf{I}^*_t,\mathbf{Y}_t;(0,\mathbf{g}^i_t,0)|\mathbf{K}_t) - D_t(\mathbf{X}_t,\mathbf{I}^{**}_{t+1},\mathbf{Y}_t;(0,\mathbf{g}^i_{t+1},0)|\mathbf{K}_t)\big] \\ & - \big[D_t(\mathbf{X}_{t+1},\mathbf{I}_t,\mathbf{Y}_t;(\mathbf{g}^x_{t+1},0,0)|\mathbf{K}_t) - D_t(\mathbf{X}_t,\mathbf{I}_t,\mathbf{Y}_t;(\mathbf{g}^x_t,0,0)|\mathbf{K}_t)\big]\big\} \\ & = \frac{1}{4}\{SYC_t - SXC_t\} + \frac{1}{4}\{SYC_t + SIC_t\} + \frac{1}{4}\{SIC_t - SXC_t\} \\ & \equiv \Delta SEC^{y,x}_t + \Delta SEC^{y,i}_t + \Delta SEC^{i,x}_t,\end{aligned}$$

and a Paasche type indicator

(11)

$$\begin{aligned}
\Delta SEC_{t+1}^{i} = & \frac{1}{4}\Big\{\big[D_{t+1}(\mathbf{X}_{t+1},\mathbf{I}_{t+1},\mathbf{Y}_{t};(0,0,\mathbf{g}_{t}^{y})|\mathbf{K}_{t+1}) - D_{t+1}(\mathbf{X}_{t+1},\mathbf{I}_{t+1},\mathbf{Y}_{t+1};(0,0,\mathbf{g}_{t+1}^{y})|\mathbf{K}_{t+1})\big] \\
& - \big[D_{t+1}(\mathbf{X}_{t+1},\mathbf{I}_{t+1},\mathbf{Y}_{t+1};(\mathbf{g}_{t+1}^{x},0,0)|\mathbf{K}_{t+1}) - D_{t+1}(\mathbf{X}_{t},\mathbf{I}_{t+1},\mathbf{Y}_{t+1};(\mathbf{g}_{t}^{x},0,0)|\mathbf{K}_{t+1})\big]\Big\} \\
& + \frac{1}{4}\Big\{\big[D_{t+1}(\mathbf{X}_{t+1},\mathbf{I}_{t+1},\mathbf{Y}_{t};(0,0,\mathbf{g}_{t}^{y})|\mathbf{K}_{t+1}) - D_{t+1}(\mathbf{X}_{t+1},\mathbf{I}_{t+1},\mathbf{Y}_{t+1};(0,0,\mathbf{g}_{t+1}^{y})|\mathbf{K}_{t+1})\big] \\
& + \big[D_{t+1}(\mathbf{X}_{t+1},\mathbf{I}_{t}^{**},\mathbf{Y}_{t+1};(0,\mathbf{g}_{t}^{i},0)|\mathbf{K}_{t+1}) - D_{t+1}(\mathbf{X}_{t+1},\mathbf{I}_{t+1}^{*},\mathbf{Y}_{t+1};(0,\mathbf{g}_{t+1}^{i},0)|\mathbf{K}_{t+1})\big]\Big\} \\
& + \frac{1}{4}\Big\{\big[D_{t+1}(\mathbf{X}_{t+1},\mathbf{I}_{t}^{**},\mathbf{Y}_{t+1};(0,\mathbf{g}_{t}^{i},0)|\mathbf{K}_{t+1}) - D_{t+1}(\mathbf{X}_{t+1},\mathbf{I}_{t+1}^{*},\mathbf{Y}_{t+1};(0,\mathbf{g}_{t+1}^{i},0)|\mathbf{K}_{t+1})\big] \\
& - \big[D_{t+1}(\mathbf{X}_{t+1},\mathbf{I}_{t+1},\mathbf{Y}_{t+1};(\mathbf{g}_{t+1}^{x},0,0)|\mathbf{K}_{t+1}) - D_{t+1}(\mathbf{X}_{t},\mathbf{I}_{t+1},\mathbf{Y}_{t+1};(\mathbf{g}_{t}^{x},0,0)|\mathbf{K}_{t+1})\big]\Big\} \\
& = \frac{1}{4}\{SYC_{t+1} - SXC_{t+1}\} + \frac{1}{4}\{SYC_{t+1} + SIC_{t+1}\} + \frac{1}{4}\{SIC_{t+1} - SXC_{t+1}\} \\
& \equiv \Delta SEC_{t+1}^{y,x} + \Delta SEC_{t+1}^{y,i} + \Delta SEC_{t+1}^{i,x}.
\end{aligned}$$

Measuring changes in outputs, investments and variable inputs, ΔSEC_t^i and ΔSEC_{t+1}^i each consist of three subcomponents. Figure 1 illustrates these different components of ΔSEC_t^i. These subcomponents can be interpreted as a finite difference approximation of the frontier's gradient measuring changes in (i) outputs *vs* variable inputs (ii) outputs *vs* investments and (iii) investments *vs* variable inputs. In practice, one simply computes (6) instead of (9).

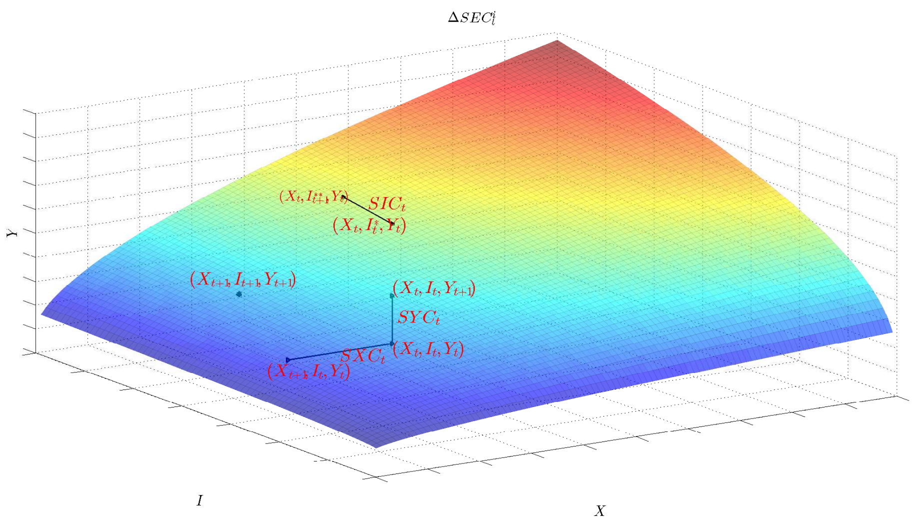

FIGURE 1. Graphical illustration of ΔSEC_t^i

4. Empirical application

4.1. **Data description.** The empirical application focuses on specialized dairy farms in South West England from 2001 – 2014. We use data from the Farm Business Survey (FBS) to this end. The FBS dataset rotates *circa* 15% of the sample on an annual basis and is unbalanced, but statistically representative. We only select specialized dairy farms that obtained an average 80% of their total revenues from milk production to ensure homogeneity of the sample. We consider seven outputs, 10 variable inputs, four quasi-fixed inputs with corresponding investments and two fixed factors. The outputs are milk, cattle meat, sheep meat, wool, pig meat, poultry and crops. The variable inputs are feed, fodder, veterinary costs, seeds, fertilizers, crop protection, electric costs, fuel, hired labor and other variable inputs. The quasi-fixed inputs and corresponding investments are breeding, buildings, machinery and improvements. In line with the literature (*e.g.*, Silva and Stefanou (2003) and Serra, Oude Lansink, and Stefanou (2011)), we set the depreciation rate as 20%, 3%, 10% and 10%, respectively. The fixed factors are land and family labor. Outputs, variable inputs, quasi-fixed inputs and corresponding investments are measured in constant 2001 £. We calculate implicit quantities per category by computing the ratio of the aggregated monetary value to the respective Törnqvist price index, which aggregates the separate price indexes. The separate price indexes are obtained from the Eurostat (2016) database. They vary thus only per year, but not per farm. Following Cox and Wohlgenant (1986), this means that differences in composition of quality are assumed to be revealed by differences in implicit quantity. This aggregation reduces dimensionality problems associated with the nonparametric approach. Land and family labor are expressed in hectares and annual working hours, respectively. Our nonparametric approach is sensitive to outliers and measurement errors. We remove influential outliers employing the approach of Banker and Chang (2006). We only keep the observations with a super efficiency score between the 5th and 95th percentile.[3] The eventual dataset contains 754 observations for a period of 14 years. We balance the sample for every subsequent pair of years to compute the LHM TFP growth, resulting in 499 LHM scores.

Table 1 shows the descriptive statistics of the variables used in the analysis.

4.2. **Practical implementation.** The smallest convex dynamic technology set $\hat{\mathcal{T}}_t$ for J farms under a variable-returns-to-scale assumption can be approximated

[3]For a concrete application of this approach to agricultural data, we refer to Ang and Kerstens (2016). To compute the super efficiency scores, we set $(\mathbf{g}^y, \mathbf{g}^x, \mathbf{g}^i) = (1, -1, 1)$ and divide the values of the observations by the respective sample means.

Variables	Dimensions	Average	Std. Dev.
Outputs	Constant 2001 £	304,837	220,037
Variable inputs	Constant 2001 £	163,139	133,516
Quasi-fixed inputs	Constant 2001 £	230,207	201,757
Investments	Constant 2001 £	38,974	47,276
Agricultural land	Hectares	110	67
Family labor	Annual working hours	4,937	1,911

TABLE 1. Descriptive statistics of variables

by:

$$\hat{\mathcal{T}}_t = \Bigg\{ \sum_{j=1}^{J} \lambda_j \mathbf{X}_{j,t} \leq \mathbf{X}_{0,t}, \tag{12a}$$

$$\sum_{j=1}^{J} \lambda_j \mathbf{Y}_{j,t} \geq \mathbf{Y}_{0,t}, \tag{12b}$$

$$\sum_{j=1}^{J} \lambda_j (\mathbf{I}_{j,t} - \boldsymbol{\delta}\mathbf{K}_{j,t}) \geq \mathbf{I}_{0,t} - \boldsymbol{\delta}\mathbf{K}_{0,t}, \tag{12c}$$

$$\sum_{j=1}^{J} \lambda_j \mathbf{L}_{j,t} \leq \mathbf{L}_{0,t}, \tag{12d}$$

$$\sum_{j=1}^{J} \lambda_j = 1. \Bigg\} \tag{12e}$$

,

We compute the time-related dynamic directional distance functions for $(a, b) \in \{t, t+1\} \times \{t, t+1\}$ by applying (1) to (12):

(13)

$$D_b(\mathbf{X}_a, \mathbf{I}_a, \mathbf{Y}_a, \mathbf{L}_a; \mathbf{g}_a | \mathbf{K}_b) = \sup \left\{ \beta \in \mathbb{R} : (\mathbf{X}_a - \beta \mathbf{g}_a^x, \mathbf{I}_a + \beta \mathbf{g}_a^i, \mathbf{Y}_a + \beta \mathbf{g}_a^y) \in \hat{\mathcal{T}}_b \right\},$$

In line with the literature (*e.g.*, Färe, Grosskopf, Noh, and Weber (2005)), we choose $\mathbf{g}^y = 1$, $\mathbf{g}^x = -1$ and $\mathbf{g}^i = 1$ and divide all variables by their respective sample mean. We compute the dynamic LHM indicator by calculating all components in (3) and (4).

4.3. **Results.** Before we compute the dynamic LHM indicator, we have tested by means of a Li test whether the distributions of the static and dynamic technologies are significantly different. We reject the null-hypothesis at $p = 0$ that the distributions are the same (see Table 2). This highlights the importance of employing dynamic rather than static directional distance functions.

H_0 hypothesis	Li statistic	p-value
$F_{\beta_{static}}(\cdot) = F_{\beta_{dynamic}}(\cdot)$ with $(\mathbf{g}^y, \mathbf{g}^x, \mathbf{g}^i) = (\mathbf{1}, \mathbf{-1}, \mathbf{0})$	14.73	0
$F_{\beta_{static}}(\cdot) = F_{\beta_{dynamic}}(\cdot)$ with $(\mathbf{g}^y, \mathbf{g}^x, \mathbf{g}^i) = (\mathbf{1}, \mathbf{-1}, \mathbf{1})$	20.22	0

TABLE 2. Model specification test based on 50 sample splits: dynamic *vs* static production technology.

Table 3 shows the general results of the average dynamic LHM indicator LHM and the decompositions into output growth, investment growth and input growth (in %). Dynamic LHM productivity has decreased by on average 2.90% *per annum (p.a.)* over the whole period. Output decline (-0.93% *p.a.*) and investment decline (-2.22% *p.a.*) have partly been offset by input decline (-0.25% *p.a.*). $LI_{t,t+1}$ is consistently the most important contributor to dynamic LHM productivity change, while $LX_{t,t+1}$ only plays a minor role. Both aggregators show negative and positive values throughout the considered period. $LY_{t,t+1}$ is consistently negative, which may be caused by the milk quota system. Notice also the importance of the additive completeness in the dynamic sense of our introduced indicator. In $2007-2010$, substantial investment growth ($+4.26\%$ *p.a.*) cancels the effect of output decline (-0.27% *p.a.*), resulting in considerable dynamic LHM productivity growth.

Table 4 shows the investment-oriented decomposition into technical change $\Delta T^i_{t,t+1}$, technical inefficiency change $\Delta TEI^i_{t,t+1}$ and scale inefficiency change $\Delta SEC^i_{t,t+1}$ (in %). The dynamic LHM indicator and components of output growth, investment growth and input growth are always well-defined (Briec and Kerstens, 2011). However, infeasibilities may arise in the components of technical change and scale inefficiency change (Ang and Kerstens, 2017b). No straightforward solutions exist to solve this problem. Following the recommendation of Briec and Kerstens (2009), we therefore simply report the proportion of infeasibilities. 36% of the scores are infeasible. This is moderately high, but in line with the number of infeasibilities for the Luenberger productivity indicator in Ang and Kerstens (2016) using the same data source. The technological frontier has on average shifted down in the investment direction (-12.36% *p.a.*), although lagging farms have on average managed to catch up in terms of technical inefficiency change ($+14.74\%$ *p.a.*). Average scale inefficiency change is slightly negative (-0.25% *p.a.*). All components fluctuate heavily throughout time. In all periods but $2010-2014$, $\Delta T^i_{t,t+1}$ and $\Delta TEI^i_{t,t+1}$ have opposite signs.

Table 5 shows the output-oriented decomposition into technical change $\Delta T^y_{t,t+1}$, technical inefficiency change $\Delta TEI^y_{t,t+1}$ and scale inefficiency change $\Delta SEC^y_{t,t+1}$ (in %). 27% of the scores are infeasible. The technological frontier has shifted down in the output direction (-0.53% *p.a.*), although lagging farms have managed to catch up in terms of technical inefficiency change ($+0.46\%$ *p.a.*). Scale inefficiency change is modestly positive ($+2.35\%$ *p.a.*). $\Delta T^y_{t,t+1}$ and $\Delta TEI^y_{t,t+1}$ have opposite signs in

TABLE 3. Average dynamic LHM indicator $LHM_{t,t+1}$ and decomposition into output growth $LY_{t,t+1}$, investment growth $LI_{t,t+1}$ and input growth $LX_{t,t+1}$ (in %)

Period	$LHM_{t,t+1}$	$LY_{t,t+1}$	$LI_{t,t+1}$	$LX_{t,t+1}$
Overall	−2.90	−0.93	−2.22	−0.25
2001 − 2004	−0.21	−1.60	+1.91	+0.53
2004 − 2007	−8.80	−2.45	−5.93	+0.43
2007 − 2010	+4.46	−0.27	+4.26	−0.46
2010 − 2014	−6.25	−0.44	−6.35	−0.55

TABLE 4. Investment-oriented decomposition into technical change $\Delta T^i_{t,t+1}$, technical inefficiency change $\Delta TEI^i_{t,t+1}$ and scale inefficiency change $\Delta SEC^i_{t,t+1}$ (in %).

Period	$\Delta T^i_{t,t+1}$	$\Delta TEI^i_{t,t+1}$	$\Delta SEC^i_{t,t+1}$
Overall	−12.36	+14.74	−0.15
2001 − 2004	−55.37	+57.81	+13.80
2004 − 2007	+39.81	−37.90	+10.17
2007 − 2010	−21.25	+31.93	−7.95
2010 − 2014	−1.89	−2.68	−6.80

all considered periods. These components fluctuate less than in the investment-oriented decomposition. On the other hand, $\Delta SEC^y_{t,t+1}$ fluctuates heavily over time and is consistently the most important contributor to the dynamic LHM indicator.

Table 6 shows the input-oriented decomposition into technical change $\Delta T^x_{t,t+1}$, technical inefficiency change $\Delta TEI^x_{t,t+1}$ and scale inefficiency change $\Delta SEC^x_{t,t+1}$ (in %). 32% of the scores are infeasible. Average technical change is negative (−1.09% *p.a.*), while technical inefficiency change (+1.37% *p.a.*) and scale inefficiency change (+4.15% *p.a.*) are on average positive. As for the output-oriented decomposition, $\Delta T^x_{t,t+1}$ and $\Delta TEI^x_{t,t+1}$ have opposite signs in all considered periods, and fluctuate less than in the investment-oriented decomposition. Also in line with the output-oriented decomposition, $\Delta SEC^y_{t,t+1}$ fluctuates heavily over time and consistently the most important contributor to the dynamic LHM indicator.

Table 5. Output-oriented decomposition into technical change $\Delta T^y_{t,t+1}$, technical inefficiency change $\Delta TEI^y_{t,t+1}$ and scale inefficiency change $\Delta SEC^y_{t,t+1}$ (in %).

Period	$\Delta T^y_{t,t+1}$	$\Delta TEI^y_{t,t+1}$	$\Delta SEC^y_{t,t+1}$
Overall	-0.53	$+0.46$	$+2.35$
$2001 - 2004$	-4.17	$+1.99$	$+27.34$
$2004 - 2007$	$+3.62$	-5.70	-29.90
$2007 - 2010$	-0.66	$+3.35$	$+10.49$
$2010 - 2014$	$+0.05$	-0.18	-5.54

Table 6. Input-oriented decomposition into technical change $\Delta T^x_{t,t+1}$, technical inefficiency change $\Delta TEI^x_{t,t+1}$ and scale inefficiency change $\Delta SEC^x_{t,t+1}$ (in %).

Period	$\Delta T^x_{t,t+1}$	$\Delta TEI^x_{t,t+1}$	$\Delta SEC^x_{t,t+1}$
Overall	-1.09	$+1.37$	$+4.15$
$2001 - 2004$	-2.56	$+2.20$	$+29.59$
$2004 - 2007$	$+2.45$	-4.04	-5.12
$2007 - 2010$	-2.60	$+5.95$	$+6.05$
$2010 - 2014$	-0.74	$+0.06$	-8.02

5. Conclusions

This paper introduces a dynamic Luenberger-Hicks-Moorsteen (LHM) productivity indicator that takes into account the adjustment costs of changing the level of quasi-fixed capital inputs. Being additively complete in the dynamic sense, the LHM indicator is decomposed into contributions of outputs, variable inputs and investments in dynamic factors. Moreover, we decompose the LHM indicator into technical change, technical inefficiency change and scale inefficiency change using an investment-, output- and input-direction. Employing a nonparametric framework, the empirical application focuses on the dairy sector in South West England over the period $2001 - 2014$.

Dynamic LHM productivity has decreased by on average 2.90% *p.a.* over the whole period. Investment decline (-2.22% *p.a.*) and output decline (-0.93%) have partly been offset by input decline (-0.25% *p.a.*). The investment component consistently plays a more important role than do the input and output components for the eventual dynamic LHM score. This underlines the importance of our dynamic specification. The output component is consistently negative, which may be caused by the milk quota system.

According to the investment-, output- as well as the input-oriented decompositions, the technological frontier has generally shifted down, while inefficient farms have generally managed to catch up. The scale inefficiency change component depends on the orientation. While the investment-oriented decomposition suggests slightly negative scale inefficiency change, the output- and input-oriented decompositions suggest positive scale inefficiency change. The absolute values of technical change and technical inefficiency change are much larger for the investment-oriented decomposition than for the output- and input-oriented decompositions, especially when one zooms in on the subperiods. One possible reason is that the distribution of investment is inherently skewed, with either large values or zero values.

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Appendix A. Dynamic LHM decomposition

The technical change component is[4]

(14)

$$\Delta T^i_{t,t+1} = \frac{1}{2}\left\{\left[D_{t+1}(\mathbf{X}_t, \mathbf{I}_t, \mathbf{Y}_t; (0, \mathbf{g}^i_t, 0)|\mathbf{K}_{t+1}) - D_t(\mathbf{X}_t, \mathbf{I}_t, \mathbf{Y}_t; (0, \mathbf{g}^i_t, 0)|\mathbf{K}_t)\right]\right.$$
$$\left. + \left[D_{t+1}(\mathbf{X}_{t+1}, \mathbf{I}_{t+1}, \mathbf{Y}_{t+1}; (0, \mathbf{g}^i_{t+1}, 0)|\mathbf{K}_{t+1}) - D_t(\mathbf{X}_{t+1}, \mathbf{I}_{t+1}, \mathbf{Y}_{t+1}; (0, \mathbf{g}^i_{t+1}, 0)|\mathbf{K}_t)\right]\right\}$$
$$\equiv \frac{1}{2}\left\{\Delta T^i_t + \Delta T^i_{t+1}\right\}.$$

Technical change $\Delta T^i_{t,t+1}$ is an arithmetic average of ΔT^i_t and ΔT^i_{t+1}. The arithmetic average is used to avoid an arbitrary choice of the observation under evaluation. Here, ΔT^i_t measures the difference in efficiency for observation $(\mathbf{X}_t, \mathbf{I}_t, \mathbf{Y}_t)$ evaluated against production frontier $t+1$ and t. An upward (downward) shift of the production frontier between t and $t+1$, indicating technical progress (regress), results in a positive (negative) difference. ΔT^i_{t+1} is similar to ΔT^i_t but evaluated for observation $(\mathbf{X}_{t+1}, \mathbf{I}_{t+1}, \mathbf{Y}_{t+1})$. Thus, technical change measures (local) shifts of the production frontier itself.

The technical inefficiency change component is

(15)

$$\Delta TEI^i_{t,t+1} = D_t(\mathbf{X}_t, \mathbf{I}_t, \mathbf{Y}_t; (0, \mathbf{g}^i_t, 0)|\mathbf{K}_t) - D_{t+1}(\mathbf{X}_{t+1}, \mathbf{I}_{t+1}, \mathbf{Y}_{t+1}; (0, \mathbf{g}^i_{t+1}, 0)|\mathbf{K}_{t+1}),$$

and measures the change between t and $t+1$ in the relative position to the production frontier. Positive (negative) values of $\Delta TEI^i_{t,t+1}$ indicate efficiency improvement (deterioration) over time: $(\mathbf{X}_{t+1}, \mathbf{I}_{t+1}, \mathbf{Y}_{t+1})$ is located closer (farther) to the $t+1$ frontier than $(\mathbf{X}_t, \mathbf{I}_t, \mathbf{Y}_t)$ was to the t frontier. Note that $\Delta TEI^i_{t,t+1}$ only measures the evolution in technical efficiency of the observation under consideration without taking into account changes of the production frontier over time.

Observe that both ΔT^o and ΔSEC^o are Fisher type indicators as they are the arithmetic average of a Laspeyres (using base period t) and a Paasche type indicator (using base period $t+1$).

[4]This section draws heavily on the respective section in Ang and Kerstens (2017a).