Generalised Nash Solution to the Puzzle of 1/2, 3/5, and 2/3 Shares in Sharecropping

Abstract

Stiglitz (1989) admits that no widely accepted theory can explicitly determine the commonly observed crop shares of 1/2, 3/5, and 2/3 in sharecropping. This is because the prevailing theories maximise the landowner's revenue instead of the profit of sharecropping farm. If all farm owners maximise their expected profits under uncertainty, Stiglitz's puzzle can be solved in rational choice model. For pure sharecropping in between fixed-wage and fixed-rent contracts, the landowner and the sharecropper are joint owners from the risk-sharing partnership perspective and certainly bargain to split the yield for their respective profit. Our model first endogenizes the bargaining powers of the landowner and sharecropper, yielding 1/2, 3/5, and 2/3 crop shares according to the generalised Nash solution that can maximise both sides' expected profits. For accuracy, we also employ the strategic game of alternating offers to achieve identical results as the Nash perfect equilibrium, thereby justifying our innovative approach.

JEL Classification: C71, D21, Q12

Keywords: expected profit; endogenous bargaining powers; generalised Nash solution; risk-sharing partnership; puzzle of 1/2, 3/5, and 2/3 crop shares

1 Introduction

This study aims to solve the puzzle of why most sharecropping contracts have the discrete crop shares of 1/2, 3/5, or 2/3 for risk-sharing. In the real world, if this key provision of sharecropping contracts cannot be determined *ex ante*, they are unlikely to be signed by anyone.

Problem. Stiglitz (1988, 120) admits that the 'agency theory cannot explain why 50-50 is the most commonly observed crop share' because any fraction in $[0,1]$ can be viewed as a qualified crop share. Moreover, the crop share $1/(1+\tau\sigma^2)$ is designed by a risk-neutral landowner for a risk-averse sharecropper (Singh 1989, 37), and has a dimensional error. Suppose the output is measured in kg and the utility u of a sharecropper is a function of their output: The output variance, σ^2, has the dimension kg^2, and the Arrow-Pratt absolute risk aversion coefficient, $\tau = -u''/u'$, has the dimension $1/kg$. Thus, $\sigma^2\tau$ has the dimension kg, and $1+\tau\sigma^2$ violates the 'dimensional homogeneity' where quantities with the same dimensions can be added or subtracted. The dimensional problem also exists in the seminal risk-sharing model developed by Stiglitz (1974, 232), in which an equilibrium crop share is $\frac{1}{3}-\frac{1}{6}kg^2$. This suggests that any pure fraction cannot be solved via risk-sharing. If a 50-50 crop share could not be specified, these contracts should not have existed. However, in India, the most common crop share has been 50-50 (Bardhan 1984, 115). In North America, when some input is shared, the 50-50 crop share dominates; when no input is shared, 50-50 falls from first to third place in crop share outcomes following 3/5 or 2/3 contracts (Allen and Lueck 2002, 90). Although numerous attempts have been made to explain this phenomenon, none have gained general acceptance (Stiglitz 1989, 22).

Related Literature. This study follows the work of Bell and Zusman (1976), who first employed *Nash* bargaining solution to determine the crop share in pure sharecropping. The results of our study, alongside those obtained by Bell and Zusman, imply that sharecropping (particularly *metayage*) is a partnership, which is a sentiment echoed by the research findings of a number of economists (Kikuchi and Hayami 1980; Murrell 1983). Only in a partnership do joint owners need to bargain over the output of their cooperation and share profit related to risk. It is the basis of *Nash* bargaining, as a two-person bargaining situation involves individuals who have an opportunity to collaborate for mutual benefit. As Nash (1950) said 'No action taken by one of the individuals without the consent of the other can affect the well-being of the other one'. This scenario requires that everyone in the group do what is best for themselves and for the group,[1] that is, both the individual and group rationalities must be satisfied.

Bell and Zusman (1976) achieved only a few crop shares close to 50-50, rather than attaining the complete 50-50 split; this was due to their model focusing solely on determining the crop share. Sharecropping actually includes a number of additional variables such as plot size, share for each crop, and cost share of the cooperating inputs (Cheung 1969, 76; Reid 1975). Bell and Zusman (1976) omitted the cooperating input, and adopted Cheung's (1968) pioneering approach of averaging the whole land (H) among sharecroppers (m) without considering land cost,[2] accepting his explanation that $\frac{sq}{h} = \frac{\partial q}{\partial h}$ indicates the following: 'the rent per acre of land equals the marginal product of land in equilibrium' where s is the landowner's crop share and $h = H / m$ is the plot size of each sharecropper. sq and $(1-s)q$ are the fractions of crop revenue q. As m decreases, h increases and $\frac{\partial q}{\partial h}$ decreases due to the diminishing marginal product; consequently, the revenue per acre of land, $\frac{sq}{h}$, also decreases. The curve, $\frac{\partial q}{\partial h}$, lies below the curve, $\frac{sq}{h}$, and neither intersect, $\frac{sq}{h} \neq \frac{\partial q}{\partial h}$. If h decreases, $\frac{\partial q}{\partial h}$ and $\frac{sq}{h}$ increase. Only when $h \to 0$, $\frac{\partial q}{\partial h} \to \infty$, and $\frac{sq}{h} \to \infty$ do $\frac{\partial q}{\partial h}$ and $\frac{sq}{h}$ intersect, $\frac{sq}{h} = \frac{\partial q}{\partial h}$. However, when $h \to 0$, $q \to 0$, neither party will sign such a contract. This approach without considering land cost cannot ensure the existence of sharecropping.

Our study also follows Binmore, Rubinstein, and Wolinsky (1986), who demonstrated how to use the power weights α ($0 \le \alpha \le 1$) and $1-\alpha$ for players 1 and 2 respectively, in the generalised *Nash* bargaining model developed by Roth, which captures the differences in player bargaining power. Each bargaining solution is characterised by $\max_{x \in X} (u_1(x_1) - d_1)^{\alpha} (u_2(x_2) - d_2)^{1-\alpha}$, where (d_1, d_2) denote the possible utility gains achievable by both parties when they fail to reach an agreement; (u_1, u_2) represents the potential utilities of both parties among the set of possible divisions, x_i, of a pie or money, M, $x_i \in X$, $X = [(x_1, x_2) | M \ge x_1 \ge 0, x_2 = M - x_1]$. When two players reach an agreement, player 1 gets $u_1 = d_1 + \alpha(M - d_1 - d_2)$ and player 2 gets $u_2 = d_2 + (1-\alpha)(M - d_1 - d_2)$, which is the 'Split-The-Difference' Rule (Muthoo 1999, 36). Binmore, Rubinstein, and Wolinsky (1986) held that, aside from the asymmetry in risk preferences, and the disagreement points that are already captured in the construction of (u_i, d_i), the α and $1-\alpha$ must be chosen to reflect the remaining asymmetries in the procedure, and the beliefs of each party in their relative bargaining power. This demonstrates that bargaining power is a vague concept, and dependent upon its sources, interrelations, and effects (Alavoine, Kaplanseren, and Teulon 2014; Eichstädt, Hotait, and Dahlen 2017). Nash (1950) was very cautious about using the term 'bargaining ability' because it suggests that players propagandise each other into misconceptions of the utilities involved. Thus, Nash assumes that the two players are highly rational, each has full knowledge of the other player's preferences, and they have equal bargaining skills, rendering any attempts at deceit meaningless.

However, it should be acknowledged that the preferences of each player are not common knowledge (Young 1993). A player is certainly likely to underplay the level of utility they have gained from a transaction if they deem it beneficial to do so. Note that due to the splitting of the given pie M , the bargaining game appears to be an experiment where the experimenter presents two white mice with a pie, and then analyses how the mice split the pie. The only difference is that the players do not use violence; in the real world, people bargain to split the object coproduced by themselves. There are ambiguities in locating d_i because of M being unrelated to d_i . One possibility is to identify d_i with the income streams accruing to the two parties during the dispute. Another possibility is to identify d_i with the income streams available to the parties if they choose the best available alternative elsewhere (Binmore, Rubinstein, and Wolinsky 1986). The third possibility is to identify d_i with the utilities that player *i* obtains in autarky (Talamàs 2020). If *Nash* bargaining is applied to splitting the crop coproduced by the landowner and sharecropper according to their own inputs, all the unrealistic assumptions and ambiguities in locating d_i disappear. We only need to assume that both owners maximise their respective expected profits, as is consistent with Nash's (1950) assumption of 'the two players are rational', and avoids his proposition that 'each has full knowledge of the other player's preferences'.

Competitive Land Rental Market. By definition, the primary feature of the competitive land rental market is that one individual is unable to affect the rental rate, r , which is the rent per acre of the land that serves the role to coordinate the lessee and the lessor. However, some economists regard the landowner's crop share s as a price-like parameter related to a *land lease* (Cheung 1968; Bardhan and Srinivasan 1971; Newbery 1974; Reid 1976). This idea is inconsistent with the following facts. First, the s , such as 1/2, 3/5, and 2/3, are explicitly used to split the crop in sharecropping. We understand the s only by how the landowner and sharecropper use it, rather than by its name 'rental share'.[3] Second, one is unlikely to see a land lease market where a landowner offers their s (1/2, 3/5, and 2/3), and a sharecropper then determines the land size rented. Third, this idea does not allow r to exist. It reflects economists' concern that 'once we permit the landlord to offer a fixed-rent contract, the Marshallian indictment of sharecropping cannot be sustained' (Otsuka, Chuma, and Hayami 1992). However, r and the fixed-rent contract exist in the real world (Cheung 1969, 72).

Stiglitz (1974) considers that s plays a role in allocating land, labour, and *risk*. He introduces the concept of 'utility equivalent contracts' – contracts that yield the same level of expected utility to the worker – and replaces the price-taking assumption of the usual competitive model with a 'utility taking' assumption. Contracts, however, are classified by the individual who claims the residual (profit). In an agricultural society, farms are owner-operated (the residual claimant and decision maker are identical). The landowner is the labourer lessee[4] and farm owner in the fixed-wage contract. The tenant is the farm owner and labourer in the fixed-rent contract (Hirshleifer, Glazer, and Hirshleifer 2005, 159). For the pure sharecropping in between the fixed-wage and fixed-rent contract, the landowner and sharecropper are joint owners from the risk-sharing partnership perspective. All farm owners desire the market value of their own inputs to be realised, and to obtain profits due to the risk they take (Knight 1921, 36). Considering that a sharecropper takes on a fraction of the risk, they should receive some profits; otherwise, they would sign a fixed-wage contract. A farm labourer (worker, sharecropper, or tenant) receives varying levels of income according to the terms of different contracts. The utility of a labourer is a function of their income in a contract. Thus, 'utility equivalent contracts' do not exist to a labourer, and a 'utility-taking' assumption is rendered void. The premise of using s to allocate land and labour, and *risk* cannot be considered to be a sound operating principle for a sharecropping model. In fact, Stiglitz (1974) still uses s to split the crop, and uses the number of sharecroppers (m) to divide the land H because he assumes that each worker possesses one 'unit' of labour and the landowner chooses the labour-land ratio to determine the contract terms.

It has been established that s is not the rental price of land, and fails to determine the optimal h of each sharecropper; so our model considers r. If r plays a role of allocating land in the competitive land rental market, s cannot play the same role. Everyone must take r rather than s as given. Sharecropping farm owner should consider the land cost per acre to maximise profit. Even under uncertainty, profits are the sole criterion by which the competitive market selects survivors (Alchian 1950). Those contracts that give rise to loss disappear.

Compared to the prevailing sharecropping models, our model is the first risk-sharing partnership in which a landowner and a sharecropper can simultaneously maximise their respective expected profits. In addition, compared to Bell and Zusman's (1976) and the current research on bargaining, our model endogenizes the bargaining powers of two players for the first time and obtains the same crop shares of 1/2, 3/5, and 2/3 as the *Nash* bargaining solution. Finally, our model can explains why the 50-50 is the dominant crop share and why labour terms are never found in sharecropping.

The article proceeds as follows. Section 2 concerns the methodology and sharecropping structures, which presents how the dominant crop share (1/2), and other crop shares (3/5 and 2/3) are derived from the signed sharecropping structures. Section 3 presents the concluding remarks.

2 Methodology and the Signed Sharecropping Structures

The contractual environment is provided as follows:

(i) Output and input markets are competitive. In a scarce economy, the land rental rate $r > 0$ (price unit/area unit*time unit), wage rate $w > 0$ (price unit/labour unit*time unit)[5], the seed or fertiliser price $\theta > 0$ (price unit/kg), and output price (normalised as one price unit/kg) are common knowledge and provided exogenously. The market guarantees full employment.

(ii) The agricultural economy consists of two typical individuals: a landowner who possesses their own homogeneous land H (measured in acre), and a landless labourer who supplies their own identical labour t (measured in labour unit). Either of these parties can sell their own identical seeds or fertiliser [6] f or buy them in the market.

(iii) The fixed wage, fixed rent, and pure sharecropping contract are freely chosen by all; they have the same contracting cost[7] and timeline of one crop year. A mixed contract is not permitted. The same crop grows under any contract.

(iv) Following Stiglitz's (1974) use of μ as a multiplicative random-variable with $E\mu = 1$, $\sigma^2 \equiv E(\mu - 1)^2 > 0$ denotes the weather uncertainty, which is not affected by labour.

(v) Technique is constant. The production function $q = \mu q(h,t,f)$ has the properties $q' > 0$, $q'' < 0$, $\frac{\partial^2 q}{\partial h \partial t} > 0$, $\frac{\partial^2 q}{\partial h \partial f} > 0$, $\frac{\partial^2 q}{\partial t \partial f} > 0$, $q(0,t,f) = 0$, $q(h,0,f) = 0$, and $q(h,t,0) = 0$. h is the size of the plot that the farm labourer works on.

We assume that farm owner maximises their expected profits under uncertainty, and that no one signs the contract if the terms cannot be specified *ex ante*. For simplicity, the model only involves a landowner and a labourer on a farm. As $E\mu q(h,t,f) = q(h,t,f)$, we omit the expectation operator E. Farm owner determines the terms by solving the following equation:

$$(1) \qquad \max_{h,t,f} q(h,t,f) - rh - wt - \theta f$$

$q(h,t,f) - rh - wt - \theta f \geq 0$ always holds,[8] that is, the market values of the individual resources involved in a farm add up to less than the market value of their product.[9] Profit motivates an owner to manage a farm, accruing to the landowner in a fixed-wage farm, and to the tenant in a fixed-rent farm. The owner bears all costs[10] and risks for the expected profit.

If a landowner provides h and shares β ($0 \le \beta \le 1$) in f, and a sharecropper provides t and shares $(1-\beta)$ in f to coproduce, then by sharing the expected profit related to risk, they are joint owners in this partnership and jointly determine the necessary terms h, f, β, and the landowner's crop share s ($0 < s < 1$) in q. The expected profit in (1) is their shared goal, which equals the total expected revenue $sq(h,t,f)+(1-s)q(h,t,f)$ minus the total input costs $rh+\theta\beta f+wt+\theta(1-\beta)f$.

Solving (1), we get $\frac{\partial q}{\partial h}=r$, $\frac{\partial q}{\partial t}=w$, and $\frac{\partial q}{\partial f}=\theta$, corresponding to Pareto optimal quantities h_2, t_2, and f_2, respectively (see point A in Figures 1, 2, 3). This implies that a landowner pays their worker the fixed wage wt_2 and a tenant pays their landowner the fixed rent rh_2. In pure sharecropping, h_2, t_2, and f_2 characterise the group rationality (*Nash* Pareto-efficiency axiom); that is, each does their best for the farm. The expected profit of a sharecropping farm is thus maximised. However, to sign the contract, both owners must determine the landowner's s in the $q(h_2,t_2,f_2)$ and their cost share β in θf_2, characterising the individual rationality that each person acts in their self-interest, that is, they maximise their own expected profits. The landowner desires the higher s, and the sharecropper the lower s, given β. This requires both sides to be involved in bargaining. $q(h_2,t_2,f_2)$ is equivalent to the M in the *Nash* program. The utility of landowners and sharecroppers are measured by their expected revenue $sq(h_2,t_2,f_2)$ and $(1-s)q(h_2,t_2,f_2)$, respectively.[11] The asymmetry arises from their input costs, $rh_2+\theta\beta f_2$ and $wt_2+\theta(1-\beta)f_2$, which can be viewed as their disagreement payoffs, equivalent to d_1 and d_2 in the *Nash* program. If either of them quits, the landowner rents out h_2 and sells their βf_2 to obtain the fixed income $rh_2 + \theta\beta f_2$; the sharecropper signs the fixed-wage contract and sells their $(1-\beta)f_2$ to get the fixed $wt_2 + \theta(1-\beta)f_2$ without taking risk. These threats are credible in competitive markets.

The *Nash* symmetry axiom says that if $d_1 = d_2$, $u_1^* = u_2^*$, which is equivalent to if $rh_2 + \theta\beta f_2 = wt_2 + \theta(1-\beta)f_2$, $sq(h_2, t_2, f_2) = (1-s)q(h_2, t_2, f_2)$. Solving for s and β , we get $s^* = 1/2$ and $\beta^* = (wt_2 - rh_2 + \theta f_2)/2\theta f_2$, and each obtains an equal expected profit $0.5q - rh_2 - \theta\beta^* f_2 = 0.5q - wt_2 - \theta(1-\beta^*)f_2$. Otherwise, one of them will not sign.

Proposition 1: In pure sharecropping, the landowner and sharecropper must stipulate the optimal plot size h_2 , optimal seed or fertiliser f_2 , landowner's cost share $\beta^* = (wt_2 - rh_2 + \theta f_2)/2\theta f_2$ in θf_2 , and 50-50 crop share split as per the *Nash* bargaining solution.

When $d_1 \neq d_2$, the 'regular' *Nash* bargaining solution (u_1^*, u_2^*) is obtained by solving $\max(u_1 - d_1)(u_2 - d_2)$. Thus, when $rh_2 + \theta\beta f_2 \neq wt_2 + \theta(1-\beta)f_2$, we solve the equivalent equation

$$(2) \qquad \max_s [sq(h_2, t_2, f_2) - rh_2 - \theta\beta f_2][(1-s)q(h_2, t_2, f_2) - wt_2 - \theta(1-\beta)f_2]$$

Solving for s given β , we obtain $s = 1/2 + [rh_2 + \theta\beta f_2 - wt_2 - \theta(1-\beta)f_2]/2q$ (Appendix A). However, the s cannot be determined *ex ante,* unless the unrealised q is already known. The regular *Nash* solution considers only the case where two players have equal bargaining power. Therefore, we extend Binmore, Rubinstein, and Wolinsky's (1986) approach using the power weights α and $1-\alpha$ to capture their relative bargaining power from $rh_2 + \theta\beta f_2$ and

$wt_2 + \theta(1-\beta)f_2$. In the real world, the larger the s that the landowner demands, the larger the $rh_2 + \theta\beta f_2$ relative to the $wt_2 + \theta(1-\beta)f_2$; conversely, the larger the $1-s$ that the sharecropper demands, the larger the $wt_2 + \theta(1-\beta)f_2$ relative to the $rh_2 + \theta\beta f_2$. Thus, we use $(rh_2 + \theta\beta f_2)/(rh_2 + wt_2 + \theta f_2)$ and $[wt_2 + \theta(1-\beta)f_2]/(rh_2 + wt_2 + \theta f_2)$ to denote the bargaining powers of the landowner and sharecropper. This is also Bell and Zusman's (1976) idea that 'the disagreement payoffs represent the bargaining power of the landlord and sharecropper'. According to the *Nash*-Roth theorem, the generalised *Nash* solution (u_1^*, u_2^*) is obtained by solving $\max(u_1 - d_1)^{\alpha}(u_2 - d_2)^{1-\alpha}$, resulting in the following equivalent equation:

$$(3) \quad \max_s [sq - rh_2 - \theta\beta f_2]^{(rh_2+\theta\beta f_2)/(rh_2+wt_2+\theta f_2)} [(1-s)q - wt_2 - \theta(1-\beta)f_2]^{1-(rh_2+\theta\beta f_2)/(rh_2+wt_2+\theta f_2)}$$

Solving (3), we obtain $s^* = (rh_2 + \theta\beta f_2)/(rh_2 + wt_2 + \theta f_2)$ (Appendix B).

Proposition 2: In pure sharecropping, both sides need to specify the optimal plot size h_2, optimal seed or fertiliser f_2, the landowner's cost-share β in the cost θf_2, and their crop share $s^* = (rh_2 + \theta\beta f_2)/(rh_2 + wt_2 + \theta f_2)$.

The s^* is independent of q. To sign the contract, s^* and β must be explicitly specified; both sides should set the simple and clear-cut s^* and then determine β from $s^* = (rh_2 + \theta\beta f_2)/(rh_2 + wt_2 + \theta f_2)$. If $s^* = 1/2$, $\beta^* = (wt_2 - rh_2 + \theta f_2)/2\theta f_2$, just as with the result in Proposition 1. Here, $\beta^* \in [0,1]$ plays the role of anti-fluctuator and usually holds even though the

h_2, t_2, and f_2 vary with r, w, and θ, respectively. This is why when input f is shared, 1/2 crop share is dominant. If rh_2, wt_2, and θf_2 lead to $\beta^* \notin [0,1]$, which violates $0 \le \beta \le 1$, both sides must shift s^* from 1/2 to 3/5, 2/3, and so on, to ensure $\beta^* \in [0,1]$. If the cost of θf_2 is borne by one side, $s^* \ne 1/2$, unless it happens to be $rh_2 = wt_2 + \theta f_2$ or $rh_2 + \theta f_2 = wt_2$. This is why when no input is shared, 50-50 crop share is likely to be dominated by 3/5 or 2/3.

Proposition 3: As long as $\beta^* \in [0,1]$ holds with the fluctuation of r, w, θ and rh_2, wt_2, θf_2, the 50-50 crop share should not change. If $\beta^* \notin [0,1]$, the dominant crop share must fall from 1/2 and shift to 3/5, 2/3, and so on, to ensure $\beta^* \in [0,1]$.

To ensure accuracy, we use the strategic game to determine sharecropping terms, where players take turns in proposing offers until they reach an agreement. The sharecropper's offer (h, t, f, β, and s) should maximise their expected profit, subject to the condition that the landowner's expected revenue does not fall below their own inputs costs, and vice versa, which forms each player's individual rationality. The equation of sharecropper and landowner, respectively, can be represented as follows:

(4) $$\max_{h,t,f,s,\beta} (1-s)q(h,t,f) - wt - \theta(1-\beta)f \quad s.t. \quad sq(h,t,f) \ge rh + \theta\beta f$$

(5) $$\max_{h,t,f,s,\beta} sq(h,t,f) - rh - \theta\beta f \quad s.t. \quad (1-s)q(h,t,f) \ge wt + \theta(1-\beta)f$$

First, we solve equation (4). From the associated Lagrangian function $L = (1-s)q(h,t,f) - wt - \theta(1-\beta)f + \lambda[sq(h,t,f) - rh - \theta\beta f]$, assuming an interior solution, the

Kuhn-Tucker conditions coincide with the ordinary first-order Lagrangian conditions. Thus, we have the necessary conditions:

(6) $\frac{\partial L}{\partial h} = (1-s)\frac{\partial q}{\partial h} + \lambda\left(s\frac{\partial q}{\partial h} - r\right) = 0$

(7) $\frac{\partial L}{\partial t} = (1-s)\frac{\partial q}{\partial t} - w + \lambda\left(s\frac{\partial q}{\partial t}\right) = 0$

(8) $\frac{\partial L}{\partial f} = (1-s)\frac{\partial q}{\partial f} - \theta(1-\beta) + \lambda\left(s\frac{\partial q}{\partial f} - \theta\beta\right) = 0$

(9) $\frac{\partial L}{\partial s} = -q(h,t,f) + \lambda q(h,t,f) = 0$

(10) $\frac{\partial L}{\partial \beta} = \theta f + \lambda(-\theta f) = 0$

We obtain $\frac{\partial q}{\partial h} = r$, $\frac{\partial q}{\partial t} = w$, and $\frac{\partial q}{\partial f} = \theta$ corresponding to h_2, t_2, and f_2, respectively (see point A in Figures 1, 2, and 3). Only when (6), (7), (8), (9), and (10) are all satisfied can the sharecropper's expected profit be maximised. Note that in (7), given h_2, f_2, and agreed β and s, $s\frac{\partial q}{\partial t} = w - (1-s)\frac{\partial q}{\partial t}$ implies $s\frac{\partial q}{\partial t}$, the marginal product revenue of labour that the landowner receives equals $w - (1-s)\frac{\partial q}{\partial t}$, which is the marginal cost of labour that the sharecropper bears.[12] In Figure 2, $s\frac{\partial q}{\partial t}$ is the difference between $\frac{\partial q}{\partial t}$ and $(1-s)\frac{\partial q}{\partial t}$, and $w - (1-s)\frac{\partial q}{\partial t}$ is the difference between the straight line BA and the curve $(1-s)\frac{\partial q}{\partial t}$. The classical models suggest that the sharecropper applies their labour only to point B, where $(1-s)\frac{\partial q}{\partial t} = w$ corresponds to t_1. However, the sharecropper cannot distinguish between $s\frac{\partial q}{\partial t}$ and $(1-s)\frac{\partial q}{\partial t}$. If the sharecropper applies more labour than t_1, they receive the marginal product revenue of labour $(1-s)\frac{\partial q}{\partial t}$ less than the prevailing wage rate w and bear the marginal cost $w - (1-s)\frac{\partial q}{\partial t}$, but $s\frac{\partial q}{\partial t}$ is larger than $w - (1-s)\frac{\partial q}{\partial t}$ because the latter is part of the former. $t > t_1$ will make q increase and then both parties will be better off. Due to $q'' < 0$, $s\frac{\partial q}{\partial t}$ decreases and $w - (1-s)\frac{\partial q}{\partial t}$ increases; hence, they all equal AC at point C where the labour level equals t_2 and the mutual benefit opportunity is exhausted. In (6), $(1-s)\frac{\partial q}{\partial h} = r - s\frac{\partial q}{\partial h}$ implies that the marginal product revenue of land that the sharecropper receives equals the marginal cost of land that the landowner bears, given t_2, f_2, and agreed β and s. The classical models never mention the $(1-s)\frac{\partial q}{\partial h}$ taken by the sharecropper, which also affects the labour supplied. In Figure 1, $(1-s)\frac{\partial q}{\partial h}$ is the difference between $\frac{\partial q}{\partial h}$ and $s\frac{\partial q}{\partial h}$; $r - s\frac{\partial q}{\partial h}$ is the difference between the r and $s\frac{\partial q}{\partial h}$ beyond point B corresponding to h_1. If the landowner provides land $h > h_1$, $(1-s)\frac{\partial q}{\partial h} > r - s\frac{\partial q}{\partial h}$ and this makes q increase until $(1-s)\frac{\partial q}{\partial h}$ and $r - s\frac{\partial q}{\partial h}$ are all equal to AC at point C where $\frac{\partial q}{\partial h} = r$. Equation (8) appears a bit complicated, so we rearrange it to obtain $s\frac{\partial q}{\partial f} - \theta\beta = \theta(1-\beta) - (1-s)\frac{\partial q}{\partial f}$. Given h_2, t_2, and an agreed β and s, $s\frac{\partial q}{\partial f} - \theta\beta$ is the net marginal product revenue that the landowner receives and $\theta(1-\beta) - (1-s)\frac{\partial q}{\partial f}$ is the net marginal cost that the sharecropper bears. In Figure 3, the sharecropper's marginal product revenue of the cooperating input f is $(1-s)\frac{\partial q}{\partial f}$, and the landowner's marginal product revenue of f, $s\frac{\partial q}{\partial f}$, is the difference between $\frac{\partial q}{\partial f}$ and $(1-s)\frac{\partial q}{\partial f}$. If the sharecropper chooses the quantity of f at point D where $(1-s)\frac{\partial q}{\partial f} = \theta(1-\beta)$ corresponding to f_1, the landowner's $s\frac{\partial q}{\partial f} - \theta\beta > 0$. The sharecropper cannot distinguish between $(1-s)\frac{\partial q}{\partial f}$ and $s\frac{\partial q}{\partial f}$. If the sharecropper chooses $f > f_1$, $s\frac{\partial q}{\partial f} - \theta\beta > \theta(1-\beta) - (1-s)\frac{\partial q}{\partial f}$. As $\theta(1-\beta) - (1-s)\frac{\partial q}{\partial f}$ is part of $s\frac{\partial q}{\partial f} - \theta\beta$, as f increases, $s\frac{\partial q}{\partial f} - \theta\beta$ decreases, and $\theta(1-\beta) - (1-s)\frac{\partial q}{\partial f}$ increases because $q'' < 0$. This makes q continue to increase until $s\frac{\partial q}{\partial f} - \theta\beta$ and $\theta(1-\beta) - (1-s)\frac{\partial q}{\partial f}$ are all equal to EC at point C, where $s\frac{\partial q}{\partial f} - \theta\beta = \theta(1-\beta) - (1-s)\frac{\partial q}{\partial f}$, namely, $\frac{\partial q}{\partial f} = \theta$ corresponding to f_2.

To maximise $(1-s)q(h,t,f)-wt-\theta(1-\beta)f$, the sharecropper derives their marginal product revenue of labour from $(1-s)q(h,t,f)$.

(11) $$\frac{d(1-s)q}{dt}=(1-s)\left(\frac{\partial q}{\partial h}\frac{dh}{dt}+\frac{\partial q}{\partial t}+\frac{\partial q}{\partial f}\frac{df}{dt}\right)=w+\theta(1-\beta)\frac{df}{dt}$$

$\frac{d(-s)q}{dt}=(1-s)\frac{\partial q}{\partial t}=w$ corresponding to t_1 (Figure 2) does not hold unless $\frac{\partial q}{\partial h}=0$ and $\frac{\partial q}{\partial f}=0$, which is impossible in a scarce economy.[13] Equation (11) tells us that the marginal labour product revenue of the sharecropper depends on h, t, f, and agreed s and β. A different plot size h or amounts of seeds or fertiliser f shift the curve $(1-s)\frac{\partial q}{\partial t}$ and $\frac{\partial q}{\partial t}$. The sharecropper must choose h_2, t_2, and f_2, i.e. $\frac{\partial q}{\partial h}=r$, $\frac{\partial q}{\partial t}=w$, and $\frac{\partial q}{\partial f}=\theta$ to make equation (11) hold. Substituting these into equation (11), we obtain the following:

(12) $$(1-s)\left(r\frac{dh}{dt}+w+\theta\frac{df}{dt}\right)=w+\theta(1-\beta)\frac{df}{dt}$$

Solving for s given β, using the closed intervals $[0,h_2]$, $[0,t_2]$, and $[0,f_2]$, respectively, we obtain the following:

(13) $$(1-s)\left(\int_0^{h_2} r\,dh+\int_0^{t_2} w\,dt+\int_0^{f_2}\theta\,df\right)=\int_0^{t_2} w\,dt+\int_0^{f_2}\theta(1-\beta)\,df$$

$s^*=(rh_2+\theta\beta f_2)/(rh_2+wt_2+\theta f_2)$. Thus, the sharecropper offers h_2, t_2, f_2, β and s^* at each turn in the contract negotiation.

Next, we solve equation (5). From the associated Lagrangian function $\zeta = sq(h,t,f) - rh - \theta\beta f + \eta[(1-s)q(h,t,f) - wt - \theta(1-\beta)f]$, we obtain the same interior solutions $\frac{\partial q}{\partial h} = r$, $\frac{\partial q}{\partial t} = w$, and $\frac{\partial q}{\partial f} = \theta$, corresponding to h_2, t_2, and f_2 (Appendix C), respectively (see Figures 1, 2, and 3). Since all the geometric analyses of the first-order conditions are similar with those of equation (4), we do not reiterate them here. To maximise $sq(h,t,f) - rh - \theta\beta f$, the landowner derives their land marginal product revenue from $sq(h,t,f)$.

(14) $$\frac{dsq}{dh} = s\left(\frac{\partial q}{\partial h} + \frac{\partial q}{\partial t}\frac{dt}{dh} + \frac{\partial q}{\partial f}\frac{df}{dh}\right) = r + \theta\beta\frac{df}{dh}$$

$\frac{dsq}{dh} = s\frac{\partial q}{\partial h} = r$ corresponding to h_1 (Figure 1) does not hold unless $\frac{\partial q}{\partial t} = 0$ and $\frac{\partial q}{\partial f} = 0$, which is impossible in a scarce economy. Similarly, the landowner must choose h_2, t_2, and f_2, i.e. $\frac{\partial q}{\partial h} = r$, $\frac{\partial q}{\partial t} = w$, and $\frac{\partial q}{\partial f} = \theta$ to make (14) hold. Substituting these into (14), we obtain the following:

(15) $$s\left(r + w\frac{dt}{dh} + \theta\frac{df}{dh}\right) = r + \theta\beta\frac{df}{dh}$$

Solving for s given β, using the intervals $[0,h_2]$, $[0,t_2]$, and $[0,f_2]$, we obtain $s^* = (rh_2 + \theta\beta f_2)/(rh_2 + wt_2 + \theta f_2)$ from the following integral equation:

(16) $$s\left(\int_0^{h_2} r dh + \int_0^{t_2} w dt + \int_0^{f_2} \theta df\right) = \int_0^{h_2} r dh + \int_0^{f_2} \theta\beta df$$

Thus, the landowner offers h_2, t_2, f_2, β, and s^* at each turn. This means that h_2, t_2, f_2, β

and s^* are both sides' best choice, and thereby comprise the unique *Nash* perfect equilibrium, regardless of who offers first. This proves *Nash*'s idea that 'the two approaches to the problem, via the negotiation, or axioms, are complementary; each helps to justify and clarify the other'.[14]

However, when signing the formal contract, both parties need not stipulate all the optimal quantities. Suppose that both sides only specify h_2, f_2, β, and s^* in the contract, if the sharecropper provided the optimal labour t_2 *ex post*, both sides would obtain their respective optimal expected revenues (see Appendix D):

$$(17)\quad (1-s^*)q(h_2,t_2,f_2)=wt_2+\theta(1-\beta)f_2+(1-s^*)[q(h_2,t_2,f_2)-rh_2-wt_2-\theta f_2]$$

$$(18)\quad s^*q(h_2,t_2,f_2)=rh_2+\theta\beta f_2+s^*[q(h_2,t_2,f_2)-rh_2-wt_2-\theta f_2]$$

Here, (17) and (18) are equivalent to the *Split-the-Difference* Rule, $u_2=d_2+(1-\alpha)(M-d_1-d_2)$ and $u_1=d_1+\alpha(M-d_1-d_2)$. If the sharecropper provided $t_1<t_2$, $(1-s^*)q(h_2,t_1,f_2)<(1-s^*)q(h_2,t_2,f_2)$, $s^*q(h_2,t_1,f_2)<s^*q(h_2,t_2,f_2)$ due to $q'>0$, both would suffer losses. This proves that h_2, f_2, β and s^* are optimal not only *ex ante* but also *ex post,* that is, they are dynamically consistent.

Proposition 4: The landowner and sharecropper need only specify the terms h_2, f_2, β, and s^* in pure sharecropping. Optimal labour t_2 can be self-enforced *ex post* by the sharecropper. Therefore, sharecropping contracts need not to specify the labour terms.

Since (h_2, f_2, β, and s^*) can maximise the expected profits of each partner, landowner and sharecropper will voluntarily sign the contracts of (h_2, f_2, β^*, 1/2), (h_2, f_2, β^*, 3/5), and (h_2, f_2, β^*, 2/3) provided $0 \le \beta^* \le 1$ is achieved according to the generalised Nash solution $s = \left(rh_2 + \theta\beta^* f_2\right)/\left(rh_2 + wt_2 + \theta f_2\right)$, s = 1/2, 3/5, 2/3.

3 Concluding Remarks

The owner of a sharecropping farm should sign contracts that maximise their own expected profits under uncertainty. Our model does not follow Cheung's (1968) or Stiglitz's (1974) paradigm as their sharecropping farms do not pursue profits. We cannot identify the farm owner based on the structure of residual claims, nor can we ascertain whether the landowner can achieve the maximum revenue or utility. If the rental rate $r > 0$, and Cheung averages land H among sharecroppers (m) to maximise $msq(h,t)$, it is likely that $msq(h,t) < rH$. This is because in mathematics, regardless of finding the maximum or minimum with a condition, the Lagrange multiplier method is the only available approach. If we replace the goal $\max msq(h,t)$ with $\min msq(h,t)$, all the first-order conditions remain unchanged, and $h = H/m \to 0$ still makes $\frac{sq}{h} = \frac{\partial q}{\partial h}$ hold in equilibrium. When $h = 0$, $msq(h,t) = 0$ is the landowner's minimum revenue relative to $rH > 0$. Indeed, extrema occur at critical points where all partial derivatives are zero. Only via diminishing marginal returns and constant marginal cost of input can we discover the maximum from critical points. Without considering land cost, $m \to \infty$ or $m \to 0$ is all possible.

Our model follows *Nash*'s bargaining game because it requires both the individual rationality of each player and the group rationality to be satisfied. This corresponds to maximising the expected profits of each individual and the sharecropping farm. Profits always come with risks. If the landowner or sharecropper is not interested in profit, they can freely choose the fixed-rent or fixed-wage contract, receiving the fixed market value of their inputs, and avoid bearing risks.

When signing pure sharecropping, the landowner and the sharecropper intend to share simultaneously profit and risk. Further, both need to jointly determine the terms (h_2, f_2, β, and s^*) to maxmise their respective profit, which involves the bargaining to split the expected yield $q(h_2, t_2, f_2)$. We naturally associate the endogenous input costs of the landowner and sharecropper ($rh_2 + \theta\beta f_2$ and $wt_2 + \theta(1-\beta)f_2$) with their respective disagreement payoffs. This enables us to step away from the ambiguities in locating d_i and the problem of M being irrelevant to d_i in the *Nash* program. Since the regular *Nash* crop share solution cannot be determined *ex ante*, our model extends Binmore, Rubinstein, and Wolinsky's (1968) approach, using power weights α and $1-\alpha$ in the generalised *Nash* solution to capture the relative bargaining power from their input costs $rh_2 + \theta\beta f_2$ and $wt_2 + \theta(1-\beta)f_2$, and obtaining the crop share $s^* = (rh_2 + \theta\beta f_2)/(rh_2 + wt_2 + \theta f_2)$. This is the sole bargaining solution, different from Bell and Zusman's (1976) indefinite bargaining solution. s^* makes the *Nash* symmetry solution 50-50 become one special case of the generalised *Nash* solution. If $rh_2 + \theta\beta f_2 = wt_2 + \theta(1-\beta)f_2$, the landowner and sharecropper have equal bargaining power and equal expected profit $0.5(q - rh_2 - wt_2 - \theta f_2)$. If $rh_2 + \theta\beta f_2 \neq wt_2 + \theta(1-\beta)f_2$, the landowner and sharecropper have unequal bargaining power and unequal expected profit of $s^*(q - rh_2 - wt_2 - \theta f_2)$ and $(1-s)^*(q - rh_2 - wt_2 - \theta f_2)$, respectively. Compared to Cheung's (1968) $(1-s)q = wt$ and Stiglitz's (1974, 227: footnote 3) constraint that each sharecropper gets exactly the utility the sharecropper can obtain elsewhere, our result shows that a sharecropper will not sign the sharecropping contract unless they can gain a fraction of expected profit. This is because the utility of the expected income wt in sharecropping is less than that of the fixed-wage wt.

Signing a sharecropping contract requires that crop share be determined *ex ante*. Our solution

s^* such as 1/2, 3/5, and 2/3 is independent of the *ex post* output q , and can be determined *ex ante* with the available information in a competitive market. Cheung's (1968) crop share $(q-wt)/q$, however, cannot be stipulated *ex ante* before the output q is realised. In an agency model, the crop share $1/(1+\tau\sigma^2)$, cannot be specified before a sharecropper's absolute risk aversion coefficient $\tau = -u''/u'$ with the dimension $1/kg$ and the risk σ^2 with kg^2 can be precisely measured. No one signs a sharecropping contract without knowing their crop share. However, Stiglitz's (1974) assumption of 'utility equivalent contracts' makes a sharecropper feel indifferent to signing any contract.

In pure sharecropping, aside from land h and labour t , there is a cooperating input such as seeds or fertiliser f , and that a tenant and a landowner bear all costs in the fixed-rent and fixed-wage farm, respectively. The worker receives a fixed-wage wt_2 , the sharecropper receives the expected revenue $wt_2 + \theta(1-\beta)f_2 + (1-s^*)(q - rh_2 - wt_2 - \theta f_2)$, and the tenant receives the expected revenue $wt_2 + \theta f_2 + (q - rh_2 - wt_2 - \theta f_2)$ after paying the landowner rh_2 . The utility of a fixed-wage, pure sharecropping, or fixed-rent contract varies by farm labourer. There are no 'utility equivalent contracts'.

The limitation of our study is that pure sharecropping is a unique example involving the bargaining problem in a risk-sharing partnership contract; we cannot guarantee that the method of endogenizing bargaining power is applicable to the generalised case. This is the future direction of the *Nash* bargaining research. Further, we need to clarify whether sharecropping farms can forgo profits. The answer to this question plays a crucial role in the firm theory wherein economists usually assume the overriding objective is to maximise profits.

Appendix A

$$N = [sq(h_2, t_2, f_2) - rh_2 - \theta\beta f_2][(1-s)q(h_2, t_2, f_2) - wt_2 - \theta(1-\beta)f_2]$$

$$\frac{d\ln N}{ds} = \frac{q(h_2, t_2, f_2)}{sq - rh_2 - \theta\beta f_2} - \frac{q(h_2, t_2, f_2)}{(1-s)q - wt_2 - \theta(1-\beta)f_2} = 0$$

$$sq(h_2, t_2, f_2) - rh_2 - \theta\beta f_2 = (1-s)q(h_2, t_2, f_2) - wt_2 - \theta(1-\beta)f_2$$

$$s = 1/2 + [rh_2 + \theta\beta f_2 - wt_2 - \theta(1-\beta)f_2]/2q$$

Appendix B

$$N = [sq - rh_2 - \theta\beta f_2]^{(rh_2 + \theta\beta f_2)/(rh_2 + wt_2 + \theta f_2)}[(1-s)q - wt_2 - \theta(1-\beta)f_2]^{(wt_2 + \theta(1-\beta)f_2)/(rh_2 + wt_2 + \theta f_2)}$$

$$\frac{dLnN}{ds} = \frac{rh_2 + \theta\beta f_2}{rh_2 + wt_2 + \theta f_2} \bullet \frac{q}{sq - rh_2 - \theta\beta f_2} + \frac{wt_2 + \theta(1-\beta)f_2}{rh_2 + wt_2 + \theta f_2} \bullet \frac{-q}{(1-s)q - wt_2 - \theta(1-\beta)f_2} = 0$$

$$s^* = \frac{rh_2 + \theta\beta f_2}{rh_2 + wt_2 + \theta f_2}$$

Appendix C

$$\zeta = sq(h, t, f) - rh - \theta\beta f + \eta[(1-s)q(h, t, f) - wt - \theta(1-\beta)f]$$

$$\frac{\partial\xi}{\partial h} = s\frac{\partial q}{\partial h} - r + \eta\left[(1-s)\frac{\partial q}{\partial h} = 0\right]$$

$$\frac{\partial\xi}{\partial t} = s\frac{\partial q}{\partial t} + \eta\left[(1-s)\frac{\partial q}{\partial t} - w\right] = 0$$

$$\frac{\partial\xi}{\partial f} = s\frac{\partial q}{\partial f} - \theta\beta + \eta\left[(1-s)\frac{\partial q}{\partial f} - \theta(1-\beta)\right] = 0$$

$$\frac{\partial\xi}{\partial s} = q(h, t, f) + \eta[-q(h, t, f)] = 0$$

$$\frac{\partial\xi}{\partial\beta} = -\theta f + \eta[\theta f] = 0$$

Thus, we obtain $\eta = 1$, $\frac{\partial q}{\partial h} = r$, $\frac{\partial q}{\partial t} = w$, $\frac{\partial q}{\partial f} = \theta$.

Appendix D

Note that $s^* = (rh_2 + \theta\beta f_2)/(rh_2 + wt_2 + \theta\beta f_2)$; thus, we obtain the expected profit of the sharecropper and landowner, respectively, if h_2, t_2, f_2, and β are given.

$$\left(1-s^*\right)q\left(h_2,t_2,f_2\right)-wt_2-\theta\left(1-\beta\right)f_2=\left(1-s^*\right)\left[q\left(h_2,t_2,f_2\right)-rh_2-wt_2-\theta f_2\right]$$

$$s^*q\left(h_2,t_2,f_2\right)-rh_2-\theta\beta f_2=s^*\left[q\left(h_2,t_2,f_2\right)-rh_2-wt_2-\theta f_2\right]$$

Thus, we derive (17) and (18):

$$\left(1-s^*\right)q\left(h_2,t_2,f_2\right)=wt_2+\theta\left(1-\beta\right)f_2+\left(1-s^*\right)\left[q\left(h_2,t_2,f_2\right)-rh_2-wt_2-\theta f_2\right]$$

$$s^*q\left(h_2,t_2,f_2\right)=rh_2+\theta\beta f_2+s^*\left[q\left(h_2,t_2,f_2\right)-rh_2-wt_2-\theta f_2\right]$$

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Endnotes

[1] This is a dialogue from the biographical film ‘A Beautiful Mind’ on John Nash, it represents the spirit of the *Nash* axiomatic approach,

[2] Stigler informed Cheung that rent is a cost of production (Cheung 1969, 42: footnote 38).

[3] Share-tenancy is not tenancy, just as the peanut is not a nut. Peanuts belong to the botanical family *Fabaceae* (the bean family).

[4] Workers are rented. It is not possible to buy the labour effort of workers (Barzel 2003, 49).

[5] Because the rent of land and the wage of labour depends on the duration of a contract, the rent per acre r and wage per unit of labour w are time-related.

[6] Seeds and fertiliser are indispensable to agricultural production.

[7] The contracting cost does not include the enforcement costs. We are only concerned with contract structures that they voluntarily sign.

[8] Jehle and Reny (2011, 221) rule out the constant returns-to-scale, to ensure the uniqueness of profit maximisation.

[9] See Alchian and Woodward (1987), who explained that the firm is a production unit that does not consume all of its output and is economically viable because of the gains in productivity from specialisation.

[10] In the fixed-wage farm, the monitoring cost is borne by the landowner (residual claimant) and unrelated to the fixed-wage of a worker. See Alchian and Demsetz (1972).

[11] Bell and Zusman (1976) used the same method to measure the utilities of the landowner and sharecroppers. The utility and production function have the same properties: continuous, strictly increasing, and strictly quasiconcave on R_+^n.

[12] This is Hsiao's (1975) explanation, which was neglected by economists.

[13] Johnson (1950) and Reid (1976) believed $\frac{\partial q}{\partial h} = 0$ to be a paradox in a scarce economy.

[14] See *Nash* (1953, 29).

Figure Legends

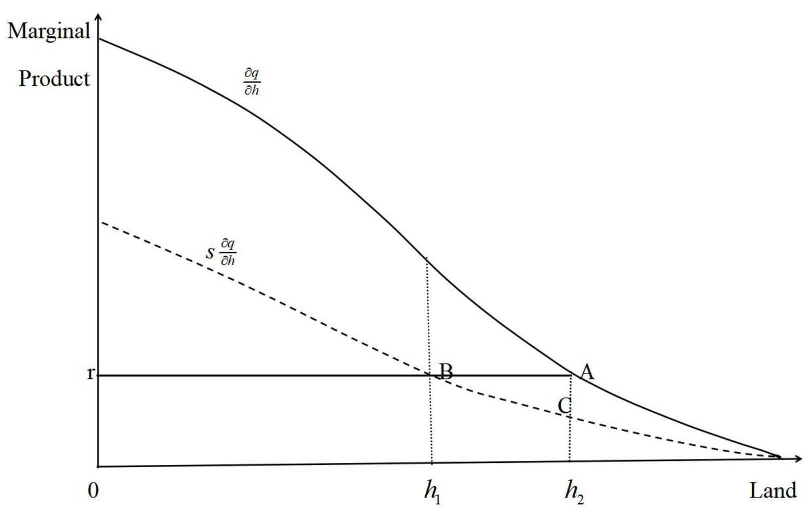

Figure 1 Determination of land size in pure sharecropping of a landowner and sharecropper

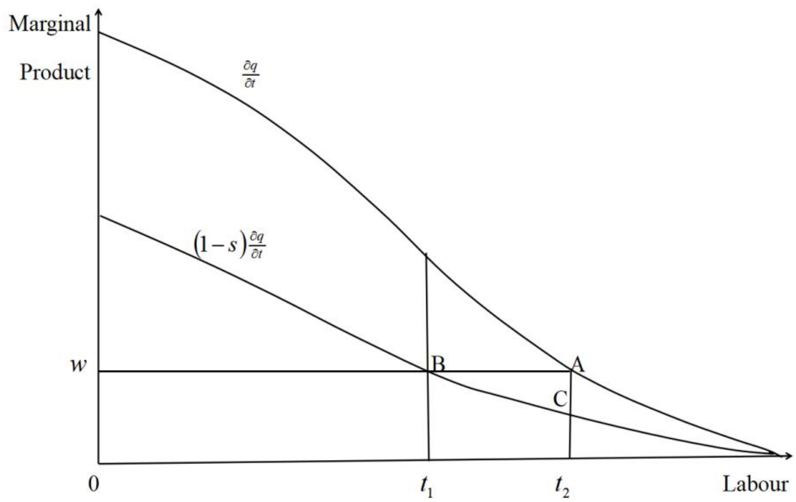

Figure 2 Determination of labour level in pure sharecropping of a landowner and sharecropper

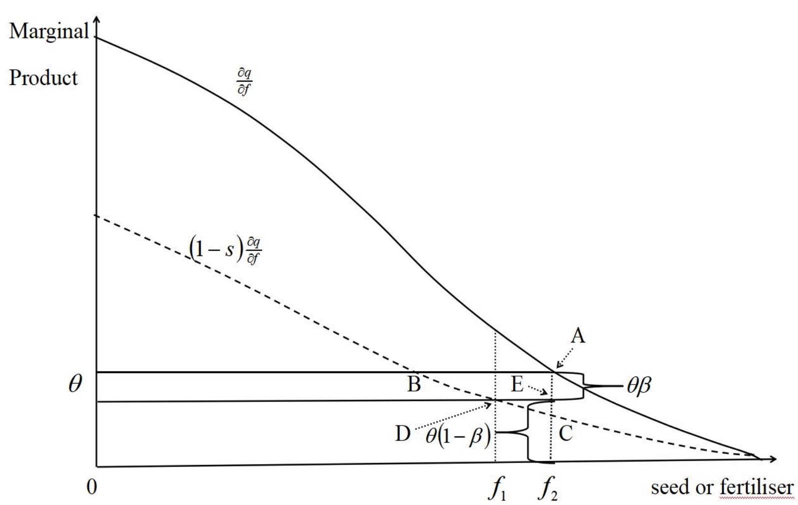

Figure 3 Determination of seed or fertilizer in pure sharecropping of a landowner and sharecropper